SynCoP 2023

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A trivial yet open synthesis problem

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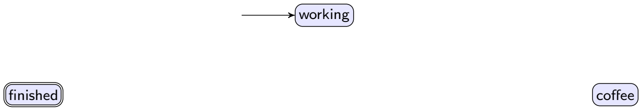
Joint problem with Didier Lime and Olivier H. Roux

Recalling timed automata (TA)

- Finite state automaton (sets of locations)



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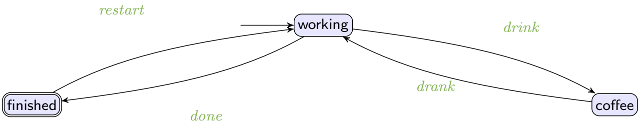
Recalling timed automata (TA)

- Finite state automaton (sets of locations and actions)



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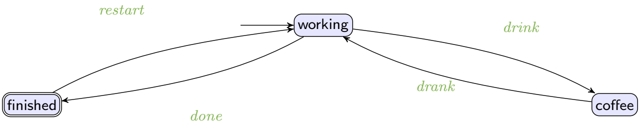
Recalling timed automata (TA)

- Finite state automaton (sets of locations and actions) augmented with a set X of clocks [AD94]
  - Real-valued variables evolving linearly at the same rate
  - Can be compared to integer constants



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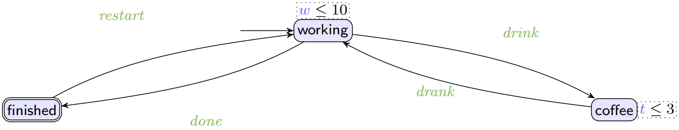
Recalling timed automata (TA)

- Finite state automaton (sets of locations and actions) augmented with a set X of clocks [AD94]
  - Real-valued variables evolving linearly at the same rate
  - Can be compared to integer constants in invariants

- Features
  - Location invariant: property to be verified to stay at a location



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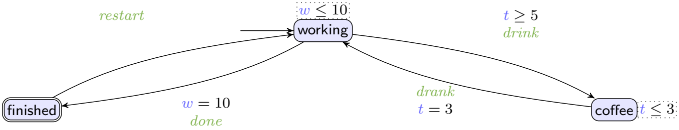
Recalling timed automata (TA)

- Finite state automaton (sets of locations and actions) augmented with a set X of clocks [AD94]
  - Real-valued variables evolving linearly at the same rate
  - Can be compared to integer constants in invariants and guards

- Features
  - Location invariant: property to be verified to stay at a location
  - Transition guard: property to be verified to enable a transition



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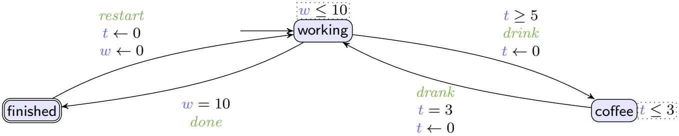
  - Transition guard: property to be verified to enable a transition
  - Clock reset: some of the clocks can be set to 0 along transitions



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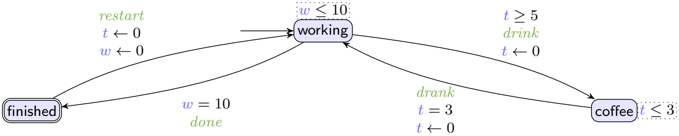
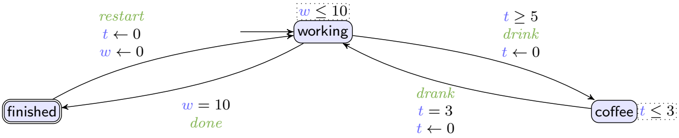


- Clock t: measuring the coffee time
- Clock w: measuring the amount of work done

[AD94] Rajeev Alur and David L. Dill. "A theory of timed automata". In: *Theoretical Computer Science* 126.2 (Apr. 1994), pp. 183–235

Recalling parametric timed automata (PTA)

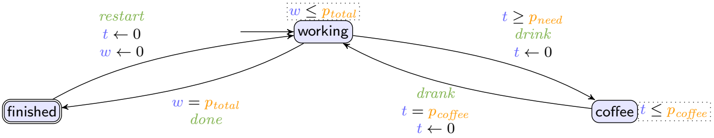
- Timed automaton (sets of locations, actions and clocks)



[AHV93] Rajeev Alur, Thomas A. Henzinger, and Moshe Y. Vardi. "Parametric real-time reasoning". In: *STOC*. ACM, 1993, pp. 592–601

Recalling parametric timed automata (PTA)

- Timed automaton (sets of locations, actions and clocks) augmented with a set P of rational-valued parameters [AHV93]
  - Unknown constants compared to a clock in guards and invariants



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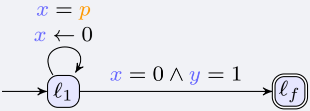
An open problem

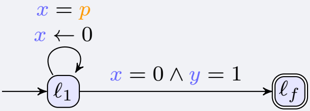
Question



What are the parameter valuations reaching ℓ_f in this PTA?

An open problem

Question



What are the parameter valuations reaching ℓ_f in this PTA?

- $p = 1$ is a solution (looping once over ℓ_1)

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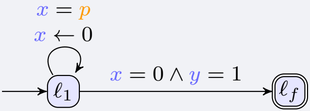

What are the parameter valuations reaching ℓ_f in this PTA?

- $p = 1$ is a solution (looping once over ℓ_1)
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An open problem

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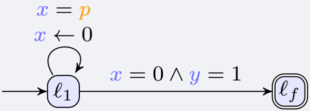

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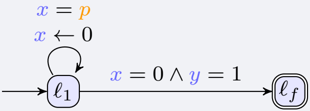

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Set of solutions:
$p \in \{\frac{1}{n} \mid n \in \mathbb{N}^{+}\}$

An open problem

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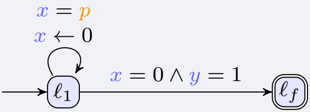

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- ...

Set of solutions:
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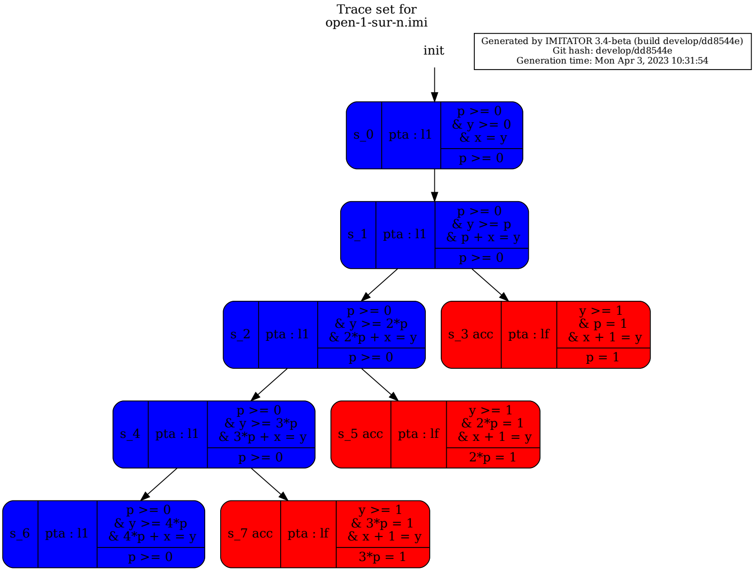
Our concrete open problem

How to compute this set automatically for this particular PTA?

Zone-based analysis diverging



What the literature says (reachability emptiness)

Most of the literature is concerned with a decision problem:

Definition (reachability emptiness)

Emptiness of the parameter valuations set for which a given location is reachable

Undecidable

- ☹ with 3 clocks and 1 parameter [Ben+15]
- ☹ with a single parametric clock + 3 non-parametric clocks [Mil00]

[Ben+15] Nikola Beneš, Peter Bezděk, Kim Guldstrand Larsen, and Jiří Srba. "Language Emptiness of Continuous-Time Parametric Timed Automata". In: *ICALP, Part II*. vol. 9135. LNCS. Springer, July 2015, pp. 69–81

[Mil00] Joseph S. Miller. "Decidability and Complexity Results for Timed Automata and Semi-linear Hybrid Automata". In: *HSCC*. vol. 1790. LNCS. Springer, 2000, pp. 296–309. ISBN: 3-540-67259-1

What the literature says (reachability emptiness)

Decidable

- ☺ with 1 clock [AHV93]
- ☺ with 1 parametric clock, n non-parametric clocks and integer-valued parameters [Ben+15]
- ☺ with 2 parametric clocks, n non-parametric clocks and 1 parameter over discrete time [GH21]
- ☺ over lower-bound/upper-bound parameters [Hun+02]

[AHV93] Rajeev Alur, Thomas A. Henzinger, and Moshe Y. Vardi. "Parametric real-time reasoning". In: *STOC*. ACM, 1993, pp. 592–601

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[GH21] Stefan Göller and Mathieu Hilaire. "Reachability in Two-Parametric Timed Automata with One Parameter Is EXPSPACE-Complete". In: *STACS*. vol. 187. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, 36:1–36:18

[Hun+02] Thomas Hune, Judi Romijn, Mariëlle Stoelinga, and Frits W. Vaandrager. "Linear parametric model checking of timed automata". In: *Journal of Logic and Algebraic Programming* 52-53 (2002), pp. 183–220

What the literature says (computation)

Goal: synthesis of all parameter valuations for which a given location is reachable

- ☺ with 1 clock [AHV93][ALM20]
- ☺ over bounded integer-valued parameters [JLR15]
- ☺ over integer-valued upper-bound parameters [BL09]

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[ALM20] Étienne André, Didier Lime, and Nicolas Markey. "Language Preservation Problems in Parametric Timed Automata". In: *Logical Methods in Computer Science* 16.1 (Jan. 2020)

[JLR15] Aleksandra Jovanović, Didier Lime, and Olivier H. Roux. "Integer Parameter Synthesis for Real-Time Systems". In: *IEEE Transactions on Software Engineering* 41.5 (2015), pp. 445–461

[BL09] Laura Bozzelli and Salvatore La Torre. "Decision problems for lower/upper bound parametric timed automata". In: *Formal Methods in System Design* 35.2 (2009), pp. 121–151

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Our open problem fits in none of these classes:

- 2 clocks
- 1 bounded rational-valued parameter

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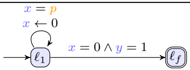
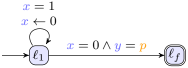
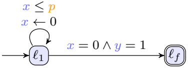
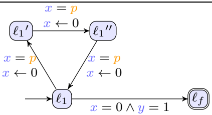
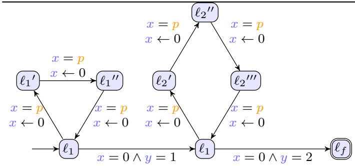
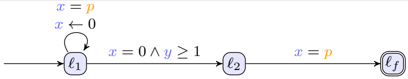
Some other concrete variants

Concrete model	Expected solutions	Class
	$p \in \{\frac{1}{n} \mid n \in \mathbb{N}^+\}$	2 clocks, 1 rational parameter
	$p \in \mathbb{N}$	2 clocks, 1 integer parameter
	$p \in (0, +\infty)$	U-PTA, 2 clocks, 1 rational parameter
	$p \in \{\frac{1}{3n} \mid n \in \mathbb{N}^+\}$	2 clocks, 1 rational parameter
	$p \in \{\frac{1}{12n} \mid n \in \mathbb{N}^+\}$	2 clocks, 1 rational parameter

Bonus: minimal-time reachability

Additional problem: what is the minimum time over all parameter valuations for which the target location is reachable?

Concrete model	Expected solutions	Class
	> 1	2 clocks, 1 rational parameter

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[AD94] Rajeev Alur and David L. Dill. “A theory of timed automata”. In: *Theoretical Computer Science* 126.2 (Apr. 1994), pp. 183–235. DOI: 10.1016/0304-3975(94)90010-8.

[AHV93] Rajeev Alur, Thomas A. Henzinger, and Moshe Y. Vardi. “Parametric real-time reasoning”. In: *STOC* (May 16–18, 1993). Ed. by S. Rao Kosaraju, David S. Johnson, and Alok Aggarwal. San Diego, California, United States: ACM, 1993, pp. 592–601. DOI: 10.1145/167088.167242.

[ALM20] Étienne André, Didier Lime, and Nicolas Markey. “Language Preservation Problems in Parametric Timed Automata”. In: *Logical Methods in Computer Science* 16.1 (Jan. 2020). DOI: 10.23638/LMCS-16(1:5)2020.

[Ben+15] Nikola Beneš, Peter Bezděk, Kim Guldstrand Larsen, and Jiří Srba. “Language Emptiness of Continuous-Time Parametric Timed Automata”. In: *ICALP, Part II* (July 6–10, 2015). Ed. by Magnús M. Halldórsson, Kazuo Iwama, Naoki Kobayashi, and Bettina Speckmann. Vol. 9135. LNCS. Kyoto, Japan: Springer, July 2015, pp. 69–81. DOI: 10.1007/978-3-662-47666-6_6.

[BL09] Laura Bozzelli and Salvatore La Torre. “Decision problems for lower/upper bound parametric timed automata”. In: *Formal Methods in System Design* 35.2 (2009), pp. 121–151. DOI: 10.1007/s10703-009-0074-0.

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[Hun+02] Thomas Hune, Judi Romijn, Mariëlle Stoelinga, and Frits W. Vaandrager. "Linear parametric model checking of timed automata". In: *Journal of Logic and Algebraic Programming* 52-53 (2002), pp. 183–220. DOI: 10.1016/S1567-8326(02)00037-1.

[JLR15] Aleksandra Jovanović, Didier Lime, and Olivier H. Roux. "Integer Parameter Synthesis for Real-Time Systems". In: *IEEE Transactions on Software Engineering* 41.5 (2015), pp. 445–461. DOI: 10.1109/TSE.2014.2357445.

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