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Parametric Schedulability Analysis of Fixed Priority Real-Time Distributed Systems

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Context: Hard Real-Time Embedded Systems

- Modern hard real-time embedded systems are distributed in nature.
- Many of them have critical timing requirements:
  - automotive systems (modern cars have 10-20 embedded boards connected by one or more CAN bus)
  - avionics systems (several distributed control boards connected by one or more dedicated networks)

- To analyse the schedulability of such systems, it is very important to estimate the (worst-case) computation times of the tasks.
- Estimating WCET is very difficult in modern architectures.

Context: Parameter Synthesis

In distributed systems, an imprecision in one timing parameter can compromise schedulability of the entire system.

$\rightsquigarrow$ Perform parametric analysis with respect to those parameters that have much variability (e.g. computation times)
Useful for:

- robustness analysis;
- design space exploration.

Outline

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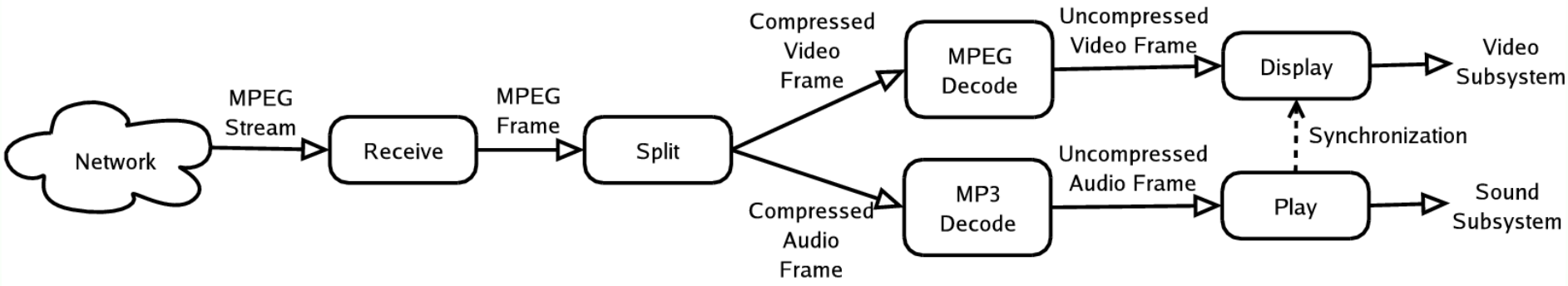
Real-Time pipelines

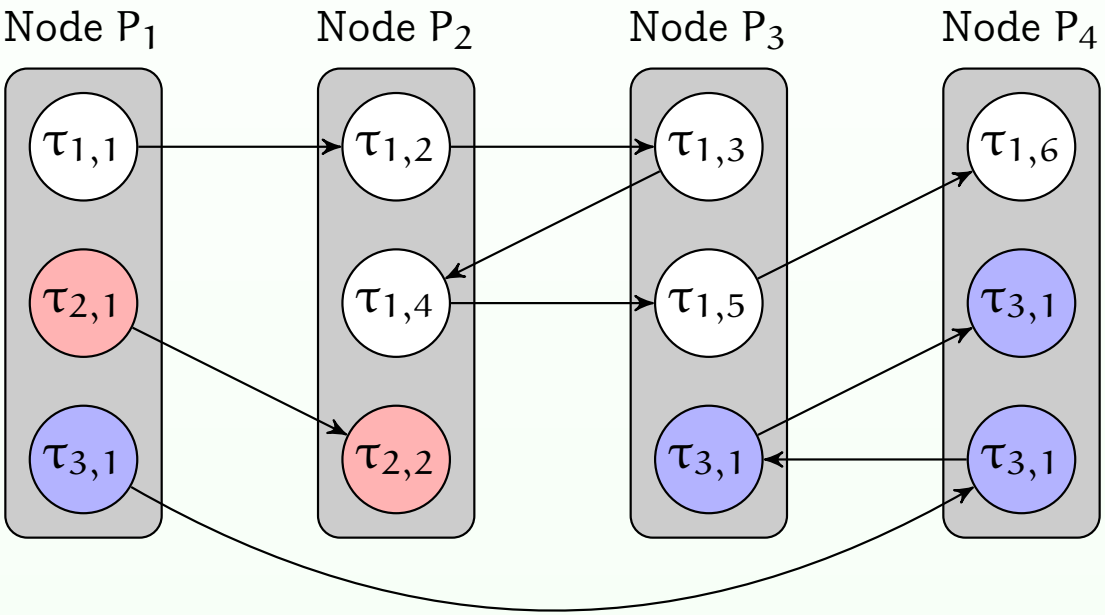
- Many real-time applications can be modeled as *pipelines* of tasks



- Executed on a distributed (or multicore) system
- Activated cyclically (periodic or sporadic)
  - In many cases, period is smaller than the end-to-end deadline
- In this paper we focus on *pipelines* (also called transactions)

Model

- A set of pipelines $\{\mathcal{P}^{(1)}, \ldots, \mathcal{P}^{(p)}\}$ distributed over m nodes
- Each pipeline $\mathcal{P}^{(i)}$ is a chain of n_i tasks $\{\tau_{i,1}, \ldots, \tau_{i,n_i}\}$
- Pipeline $\mathcal{P}^{(i)}$ has an end-to-end (E2E) deadline D_i and period T_i

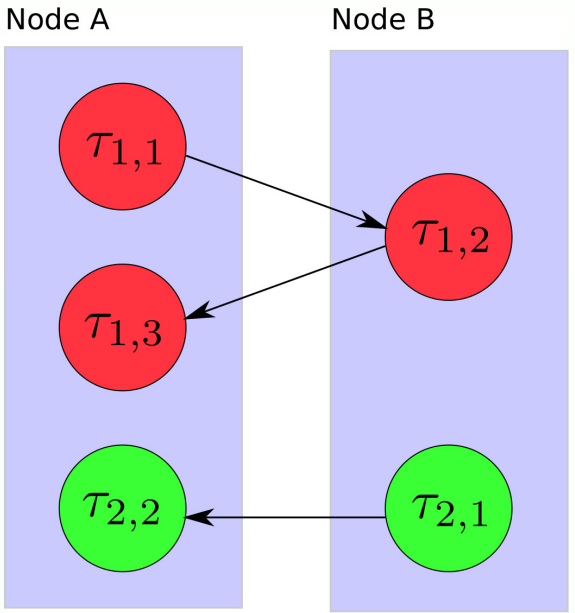


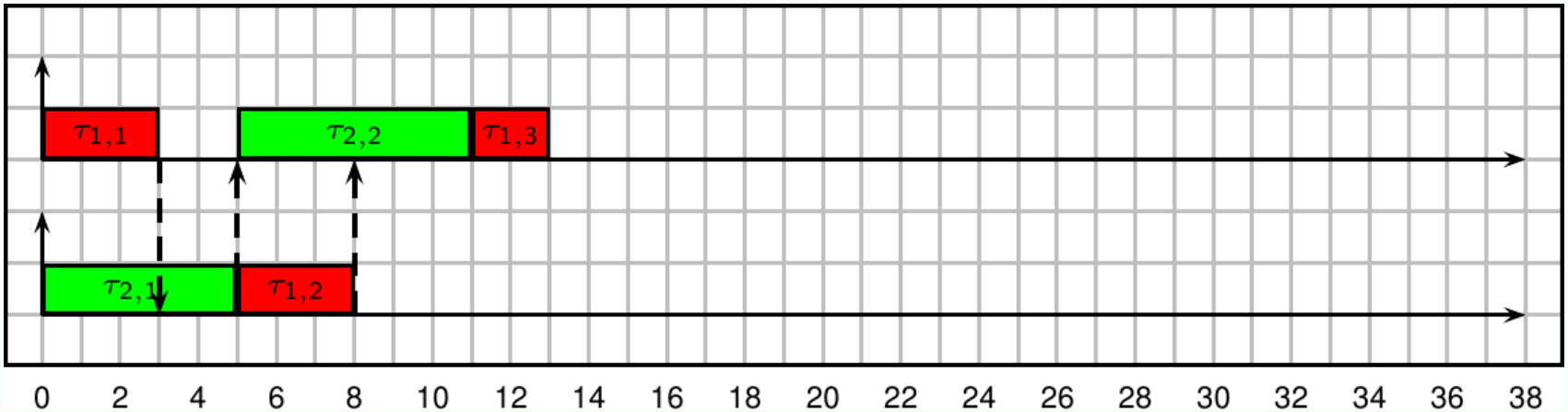
- Scheduling Problem: Guarantee that all pipelines complete before their E2E deadlines

Activations, jitter, deadline

- An example

Task	Per.	E2E	Comp.	Resp.	Jitter	prio
$\tau_{1,1}$	15		3	3	0	HI
$\tau_{1,2}$	-		3	8	0-2	LO
$\tau_{1,3}$	-	15	2	13	3	LO
$\tau_{2,1}$	12		5	6	0	HI
$\tau_{2,2}$	-	12	6	?	0-1	ME





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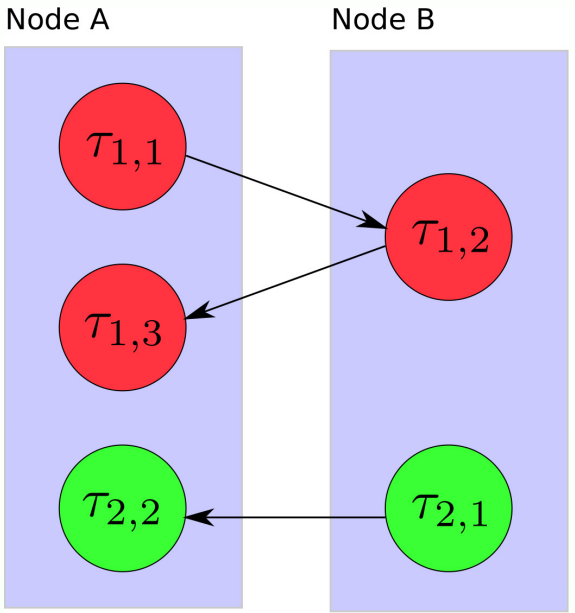

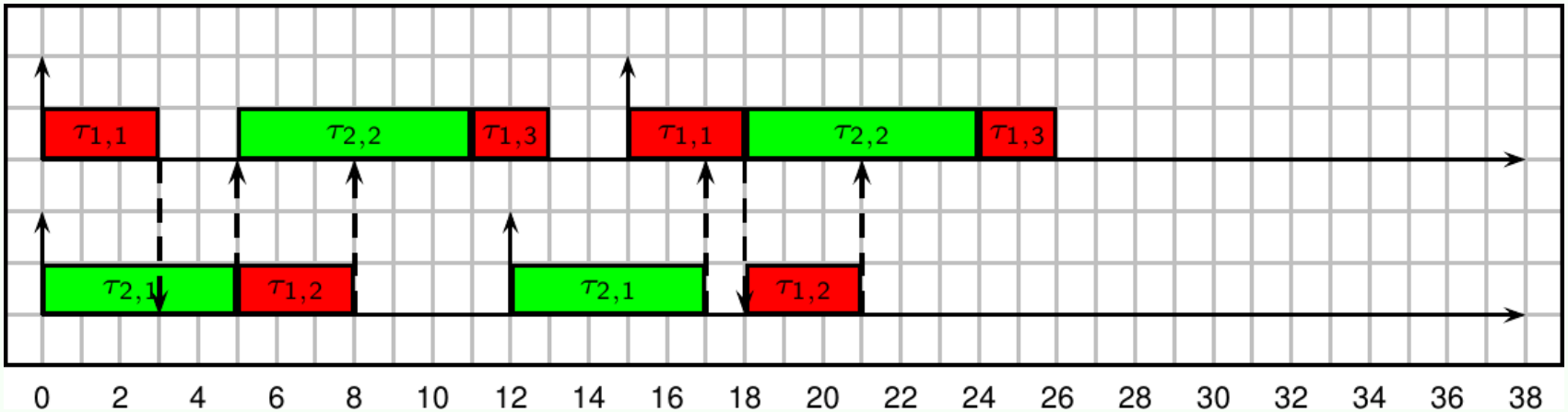

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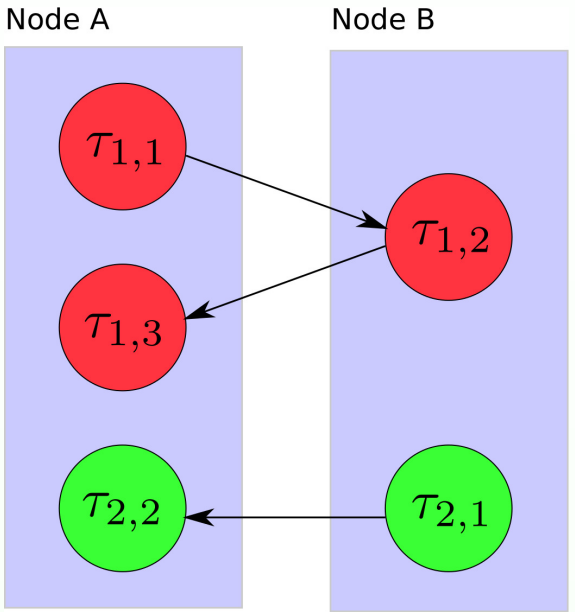

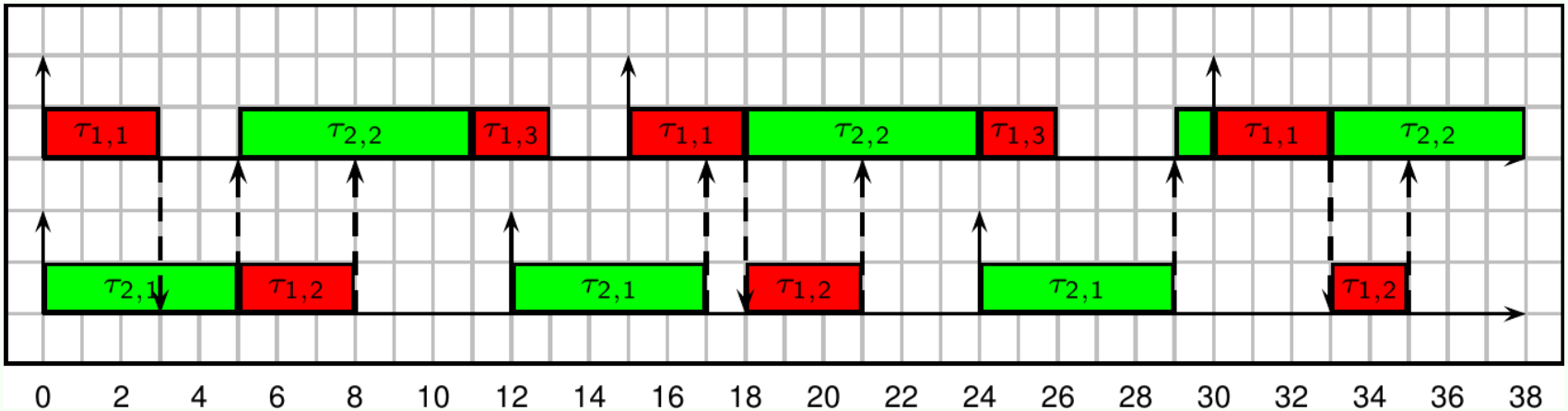

Model of the Network

- Tasks executing on processors are scheduled by Full Preemptive Fixed Priority Scheduler (FPFPS)
- Tasks send messages over the network to signal finishing of instance and activating the subsequent task
- Messages are scheduled over the network according to Non-Preemptive Fixed Priority
  - In the industrial domain, the CAN Bus features such a policy

Outline

Parametric analysis

- Previous work:
  - Holistic analysis in fixed priority [Tindell and Clark, 1994]
  - Event stream and Real-Time Calculus models
- Compute upper bounds on response times of tasks by computing the fixed point of a set of non-linear recursive equations
- Parametric analysis:
  - For which values of the computation times the system is schedulable?
  - Useful for robustness analysis and exploration of design space
- How it can be done:
  - Repeat holistic analysis for a range of parameters
  - But does not scale very well for > 2 parameters

Goal

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Provide a full-parametric analysis of a distributed real-time system with respect to the computation times of the tasks

- Two methodologies of analysis
  1. Use an extension of the holistic analysis to make it parametric
  2. Use Parametric timed automata

Outline

Parametric model

- A task is identified by three parameters:
  - C_i is the worst-case computation time (or worst-case transmission time, in case it models a message)
  - R_i is the task worst-case response time, i.e. the worst case finishing time of any task instance relative to the activation of its pipeline.
  - J_i is the task worst-case activation jitter, i.e. the greatest time since its activation that a task must wait for all preceding tasks to complete their execution
- The only one of interest is the computation time

Formula for single processor

Theorem

A task τ_i *is schedulable if*
$\forall h = 1, \ldots, \frac{H_n}{T_i}, \quad \exists \mathbf{n} \in \mathbb{P}^{i-1}((h-1)T_i + D_i)$ *such that*

- $B_i + (h-1)C_i + \sum_{j=1}^{i-1} n_j C_j \leq n_l T_l - J_l \quad \forall l = 1, \ldots, i-1$;
- $B_i + (h-1)C_i + \sum_{j=1}^{i-1} n_j C_j \leq (h-1)T_i + R_i - C_i - J_i$;
- $R_i \leq D_i$ *and* $B_i \leq C_j - 1$ *for all* $j > i$.

where $\mathbf{n}$ *is a vector of* $i-1$ *integers,* $\mathbb{P}^{i-1}(D_i)$ *is the set of scheduling points, and* B_i *is the blocking time due to non-preemptive scheduling.*

- This is a disjunction (OR) of conjunctions of inequalities (AND)
- The final polyhedron is non-convex

Distributed system

1. For each processor and network, we build the constraint system corresponding to the theorem.
   - Notice that the set of constraints for the individual single processor systems are independent of each other (because they are constraints on different tasks).

2. For each pipeline $\mathcal{T}^a$:
   - two successive tasks $\tau_{a,i}$ and $\tau_{a,i+1}$ must fulfill the constraint $R_{a,i} \leq J_{a,i+1}$;
   - for the initial task we impose $J_{a,1} = 0$.

3. After integration, all parameters $R_{a,i}$ and $J_{a,i}$ can be eliminated by projection

Advantages and Drawbacks

Advantage

- This method directly generates a set of constraints that defines the space of parameters C_i;

Drawbacks

- Since the equations give upper bounds, the space describes a subset of the real-space of constraints
- If too many parameters are specified or too many scheduling points, complexity may explode

In the experimental part, we will evaluate the scalability of this approach

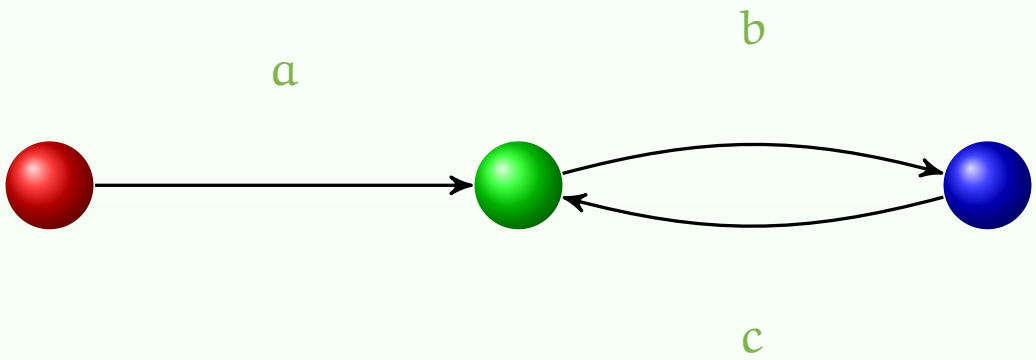
Outline

Timed Automaton

- Finite state automaton (sets of locations)

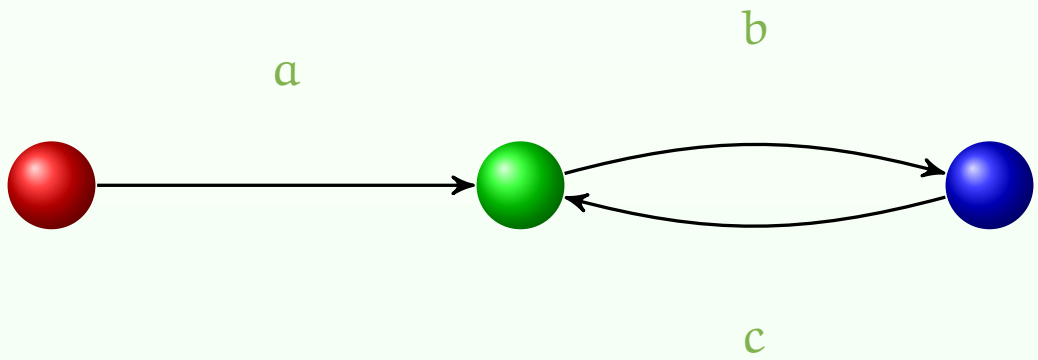
Timed Automaton

- Finite state automaton (sets of locations and actions)



Timed Automaton

- Finite state automaton (sets of locations and actions) augmented with
  - A set X of clocks (i.e., real-valued variables evolving linearly at the same rate [Alur and Dill, 1994])



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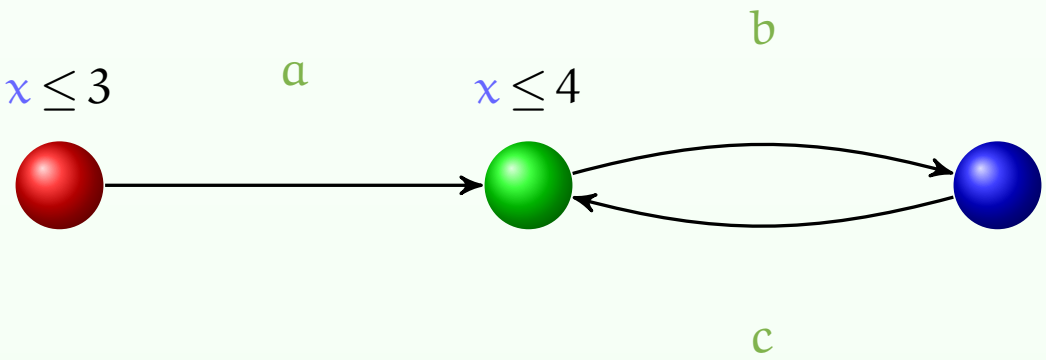
- Features
  - Location invariant: property to be verified to stay at a location



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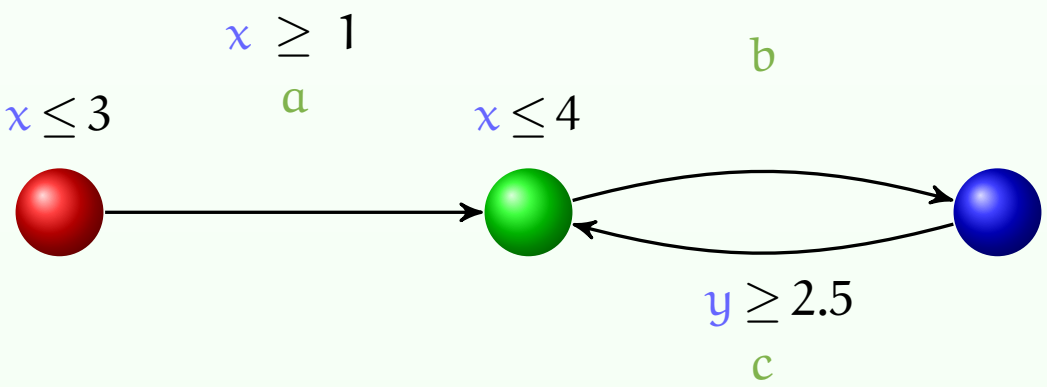

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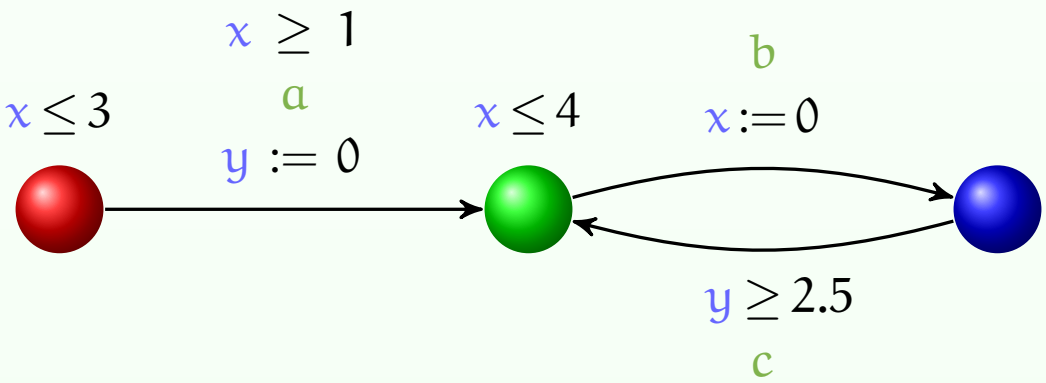
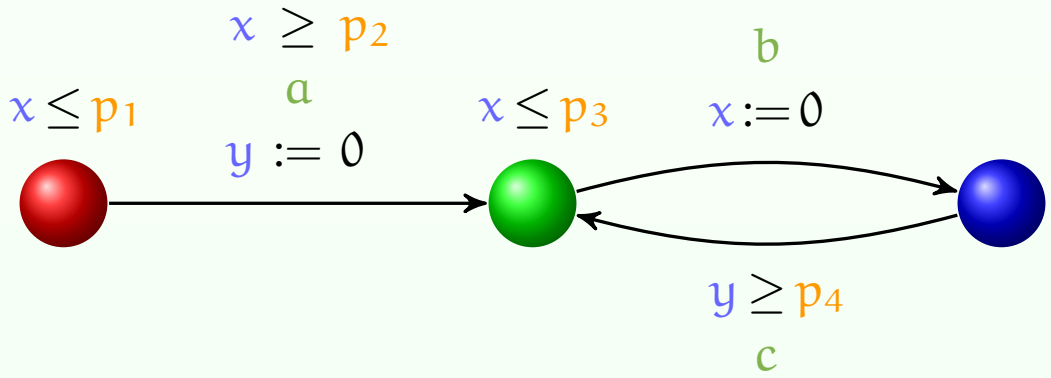
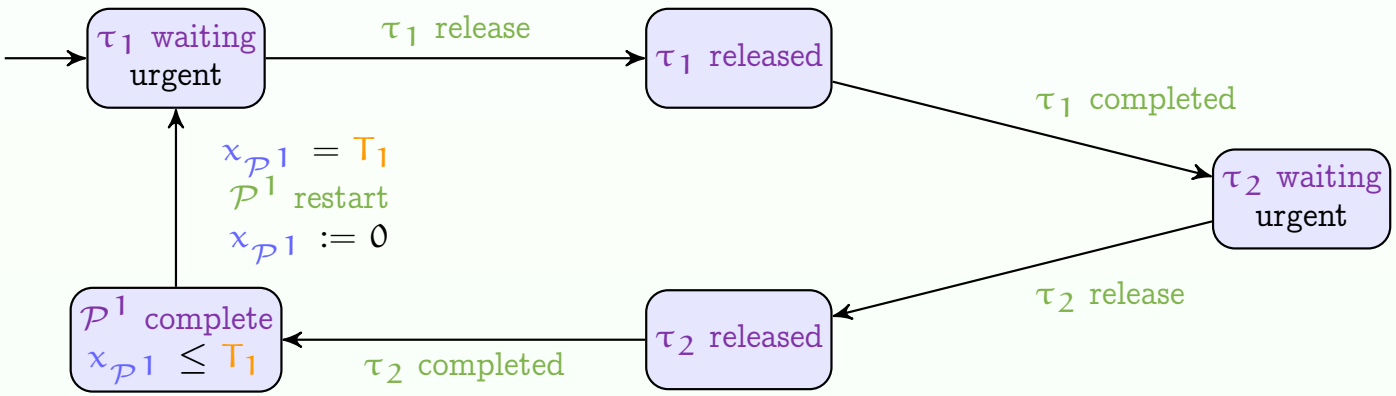


Parametric Timed Automaton (PTA)

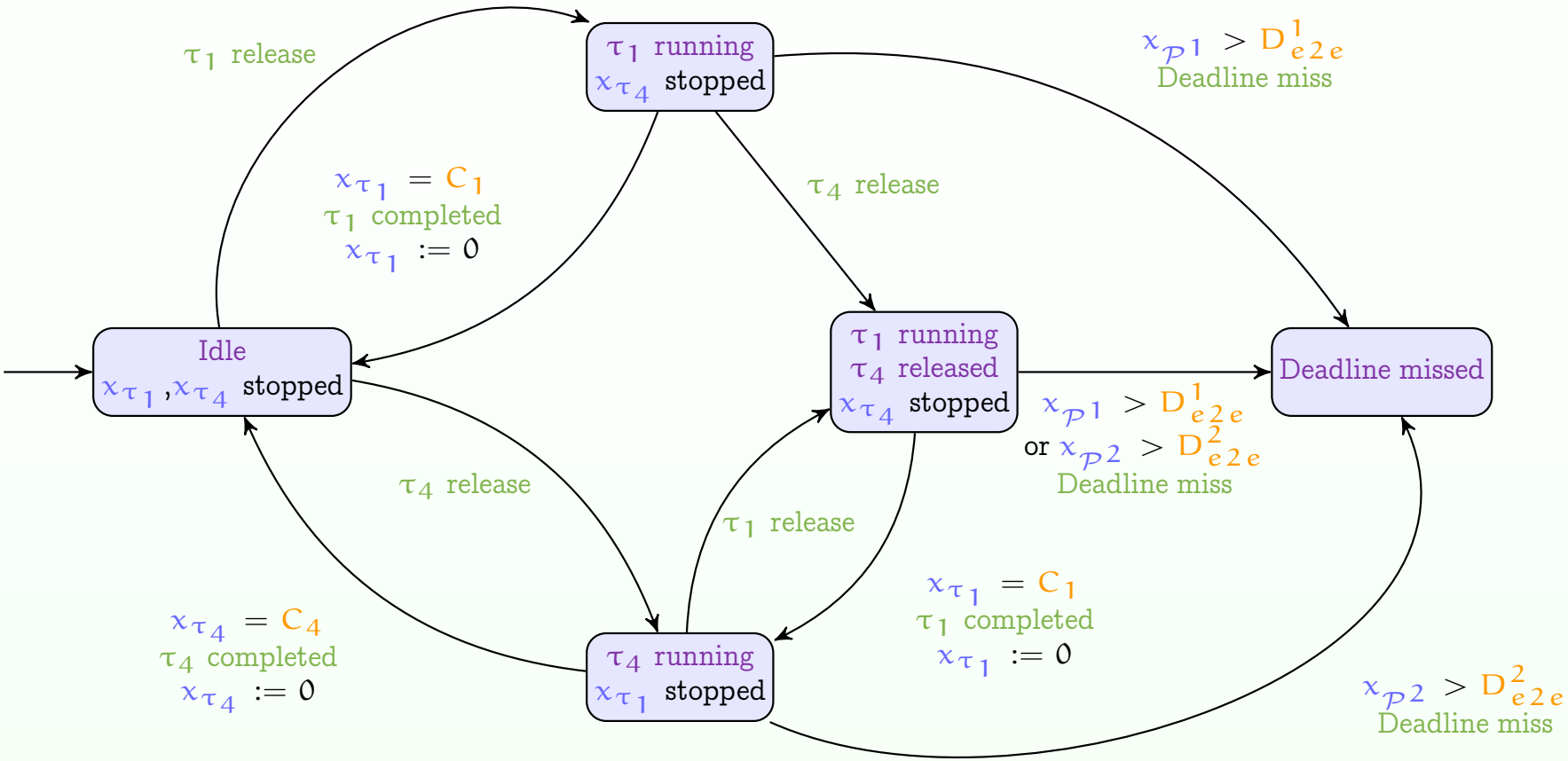
- Finite state automaton (sets of locations and actions) augmented with
  - A set X of clocks (i.e., real-valued variables evolving linearly at the same rate [Alur and Dill, 1994])
  - A set P of parameters (i.e., unknown constants), used in guards and invariants [Alur et al., 1993]
- Features
  - Location invariant: property to be verified to stay at a location
  - Transition guard: property to be verified to enable a transition
  - Clock reset: some of the clocks can be set to 0 at each transition



Modeling a Task / Pipeline



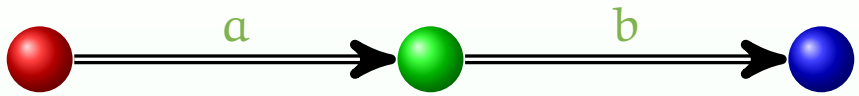
Modeling the Fixed Priority Scheduler (Preemptive)



Actually a PTA extended with stopwatches [Adbeddaïm and Maler, 2002]

Traces

- Trace over a PTA: time-abstract path
  - Finite alternating sequence of locations and actions



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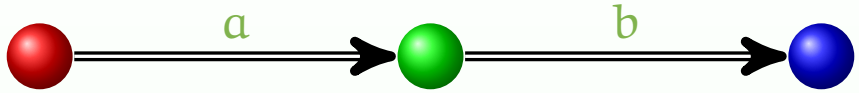
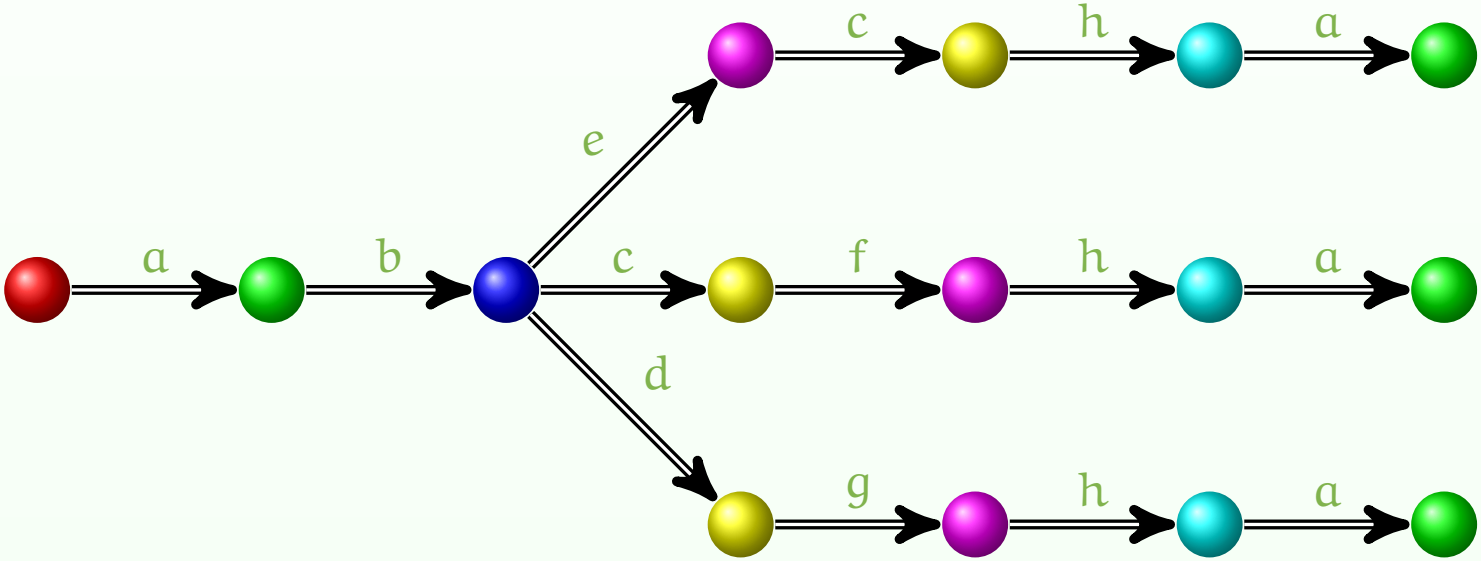


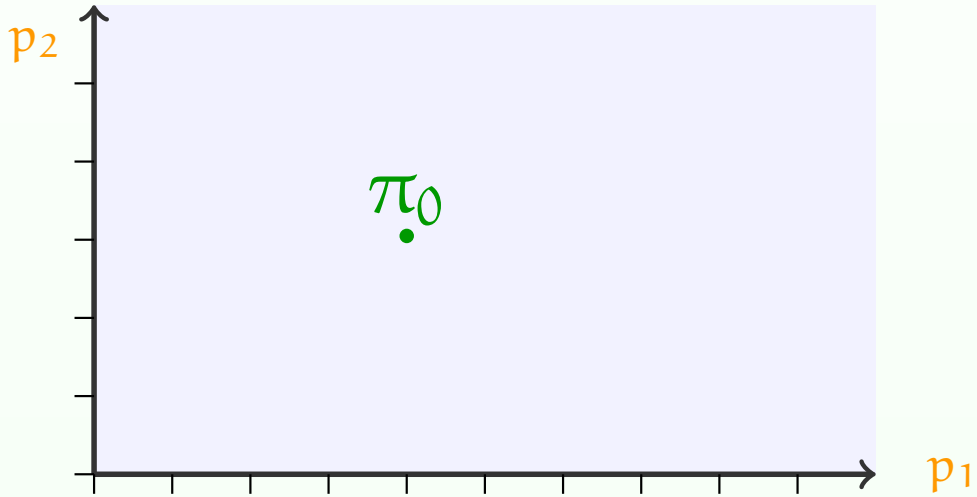
- Trace set of a PTA: set of all its traces



The Inverse Method *IM*

- Algorithm for the synthesis of parameters in PTA
  [A., Chatain, Encrenaz, Fribourg, 2009]

- Takes as input a reference parameter valuation π_0, and outputs a constraint K generalizing this reference valuation



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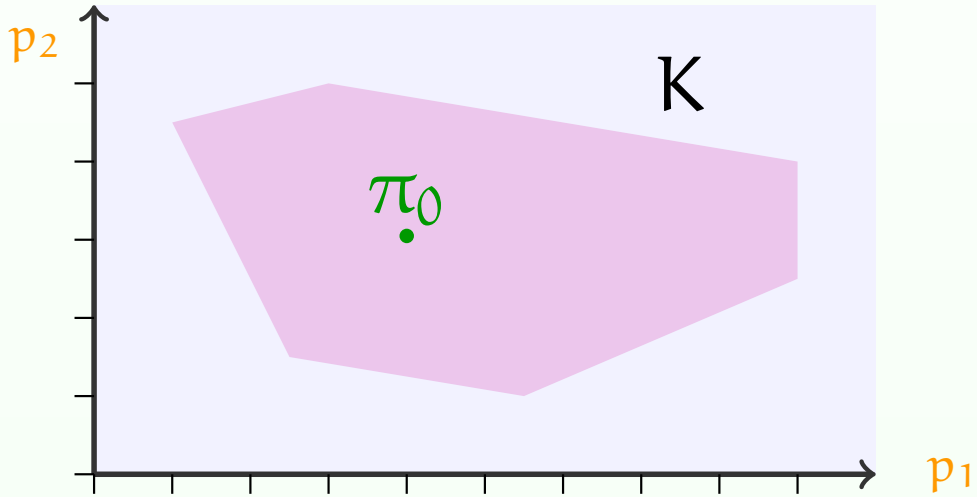

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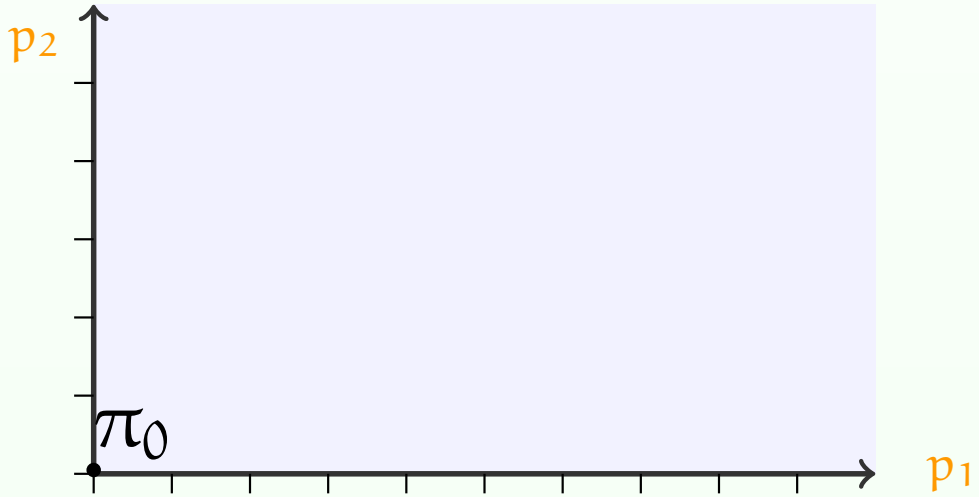
The Inverse Method: Implementation in IMITATOR

- IMITATOR 2.6 [A., Fribourg, Kühne, Soulat, 2012]
  - "Inverse Method for Inferring Time AbstracT BehaviOR"
- Features
  - Synthesis of timing parameters
  - With respect to various properties (using observer patterns)
  - Numerous analysis options
- Inside the tool
  - 10,000 lines of OCaml code
  - Uses the PPL library for operations on polyhedra [Bagnara et al., 2008]
- Use it!
  - Open source: Available under the GNU-GPL license
  - Now integrated in the *CosyVerif* platform [AHHKLLP13]

`http://www.lsv.ens-cachan.fr/Software/imitator/`

Behavioral Cartography of a Model

- Principle: Tiling of the parametric space by iteratively calling IM [A. and Fribourg, 2010]
- Each tile has the same trace set
- Good or bad depending on the presence or not of deadline misses
- Good values for the parameters: union of all regions



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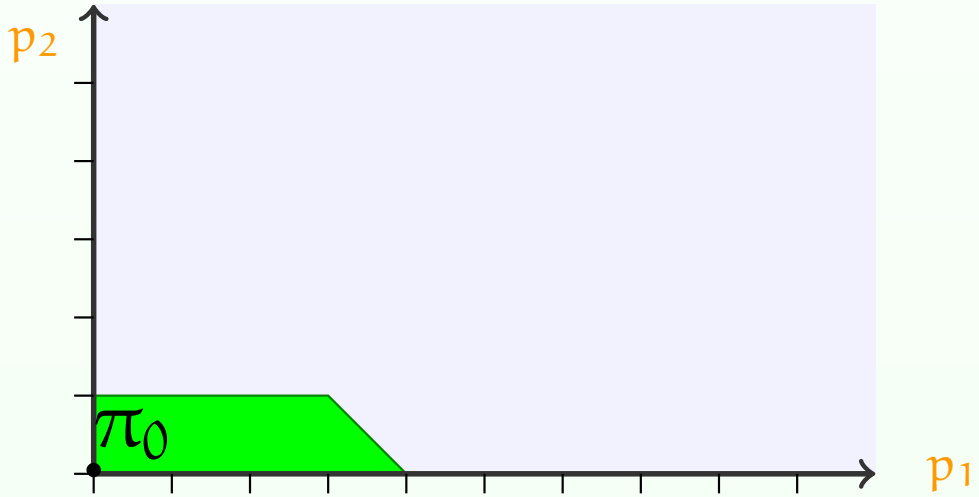

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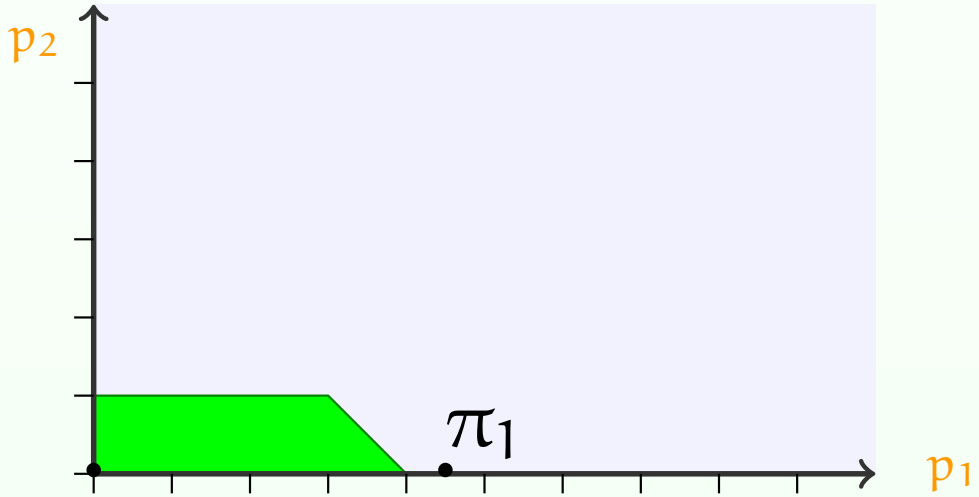

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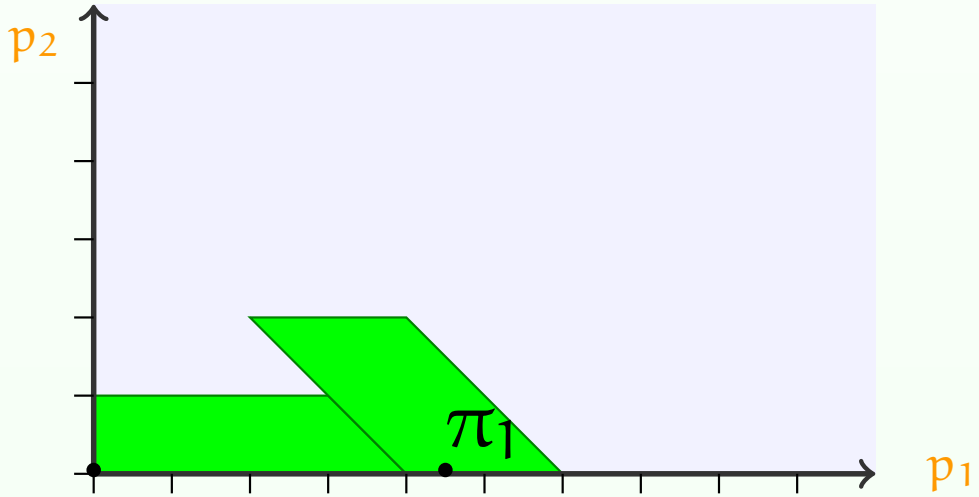

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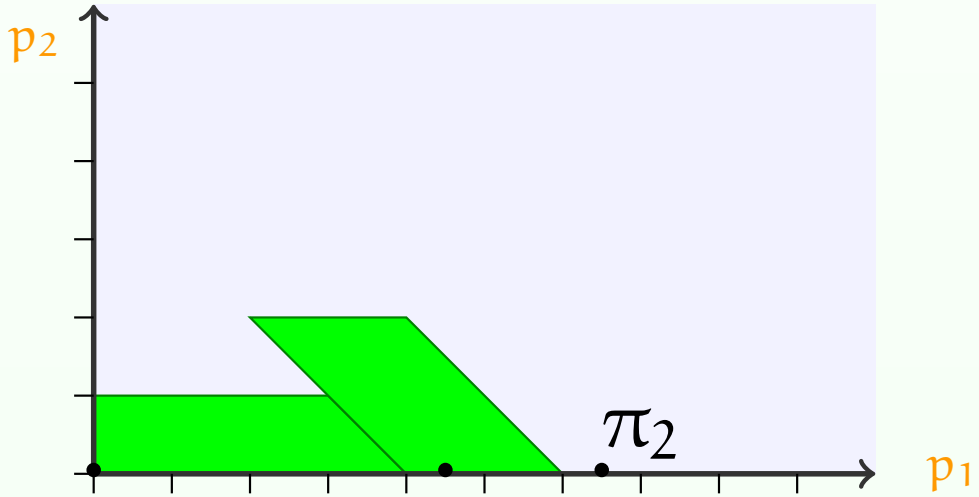

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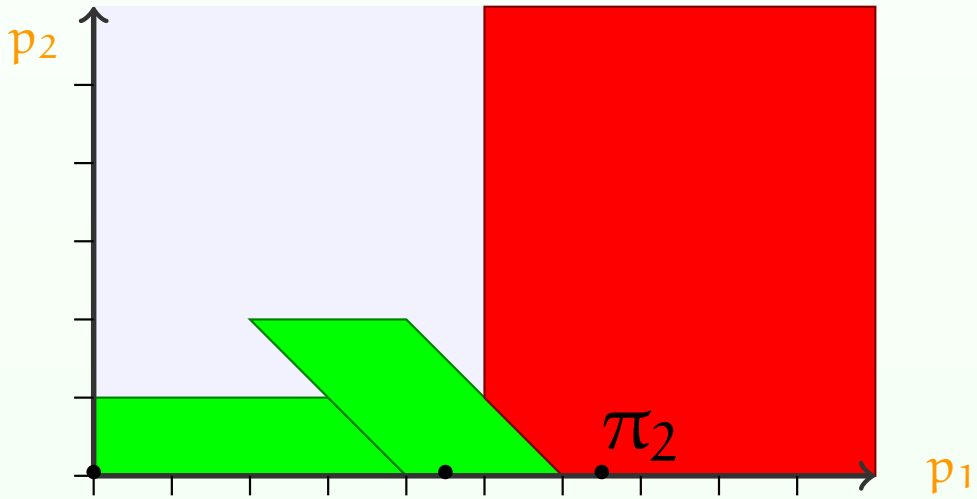

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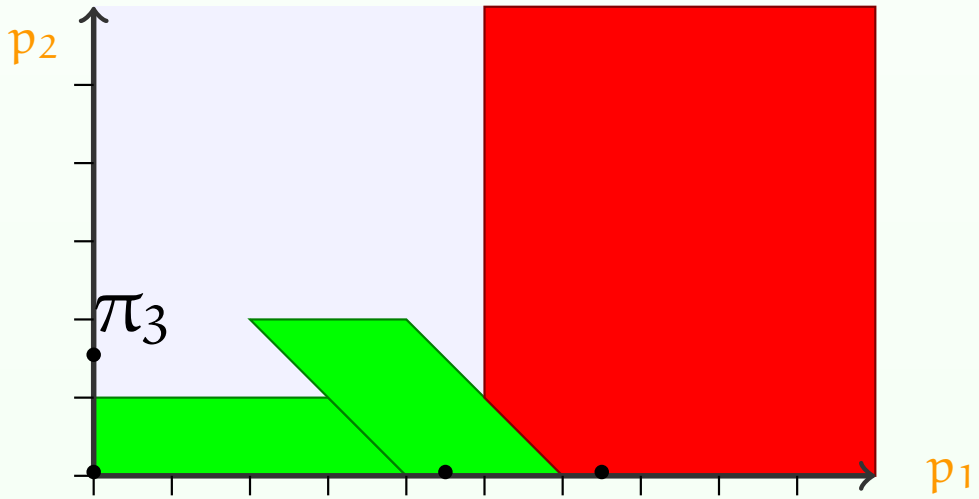

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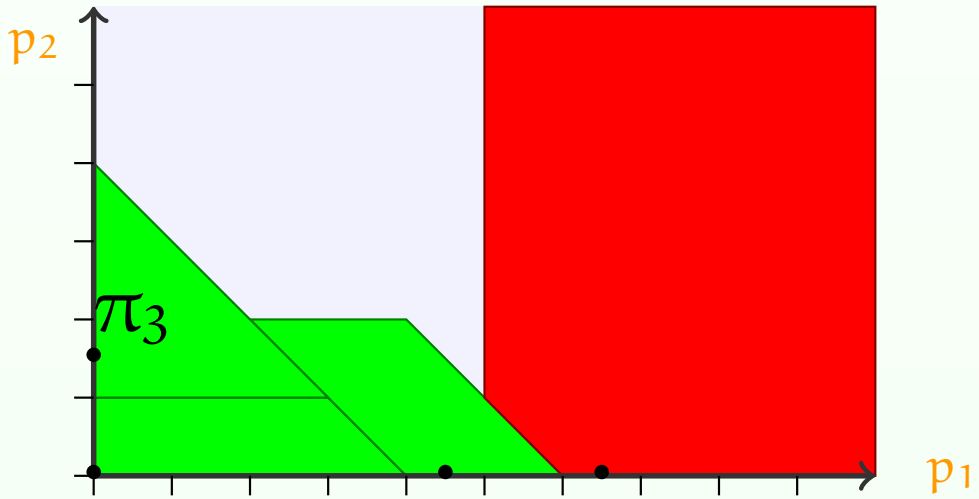

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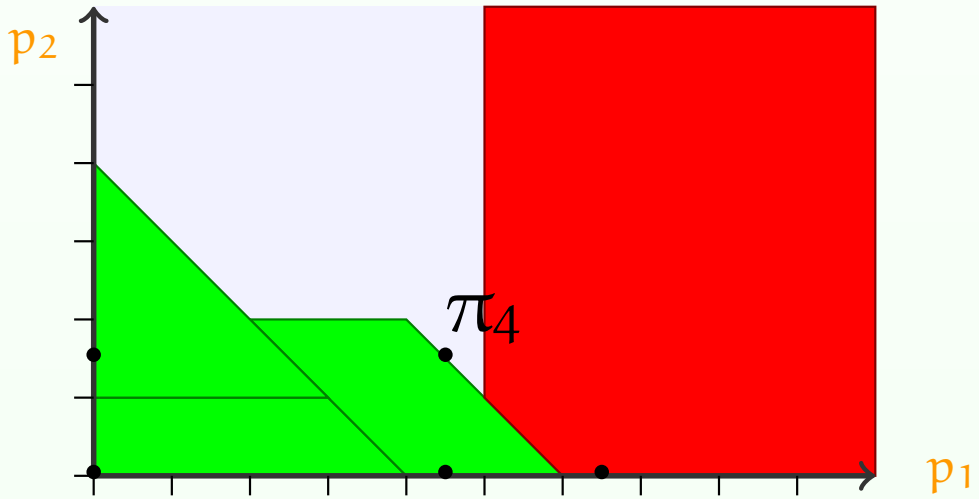

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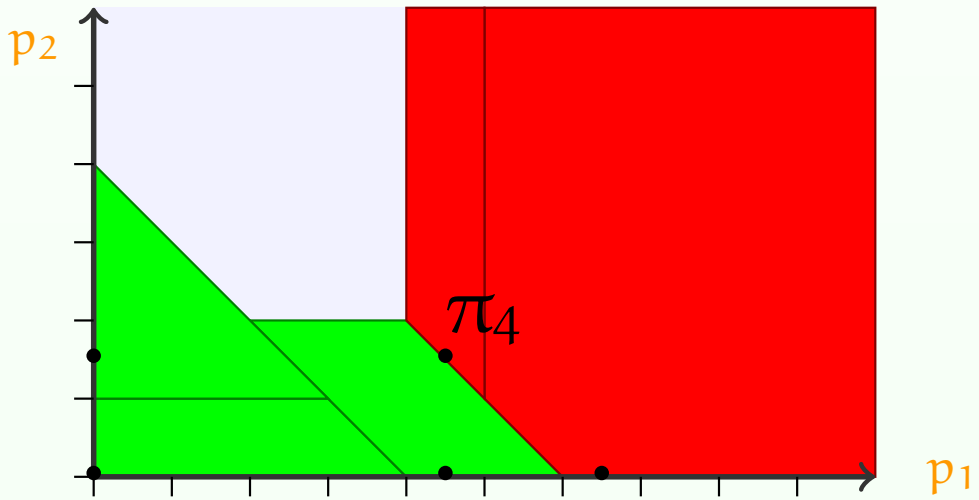

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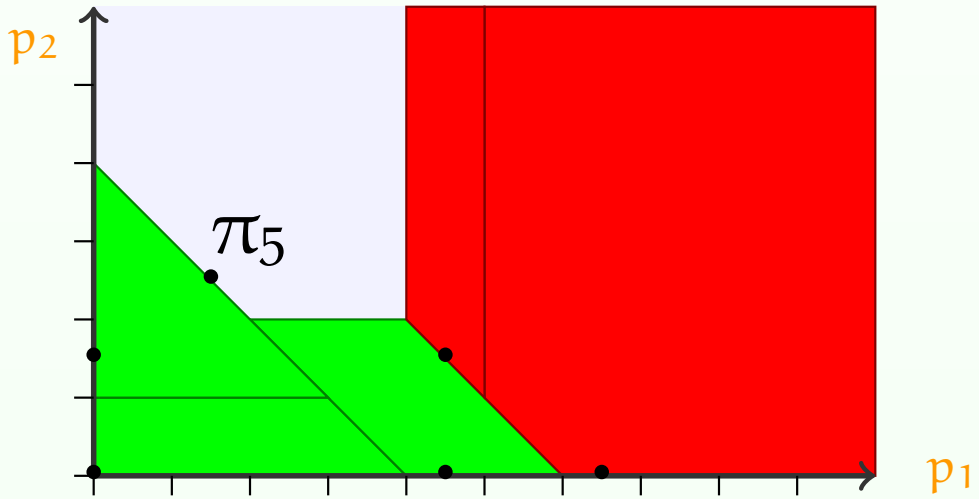

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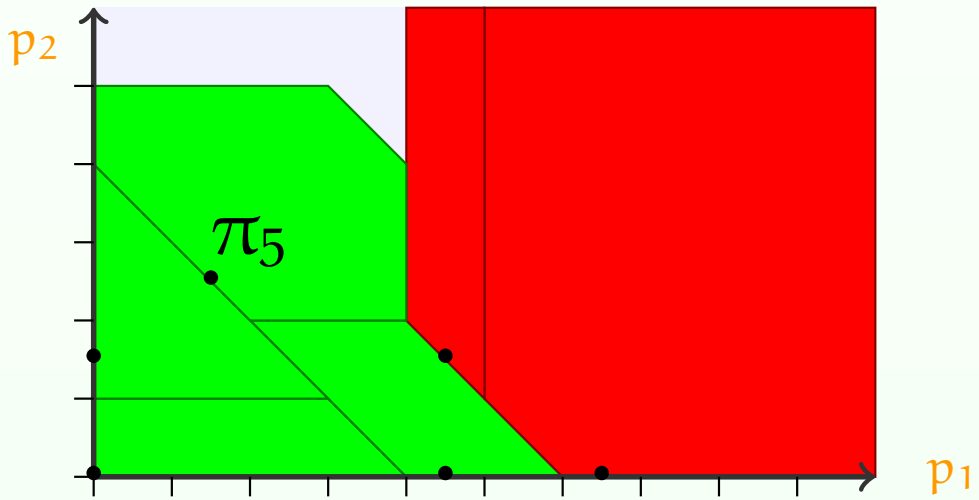

Behavioral Cartography of a Model

- Principle: Tiling of the parametric space by iteratively calling IM [A. and Fribourg, 2010]
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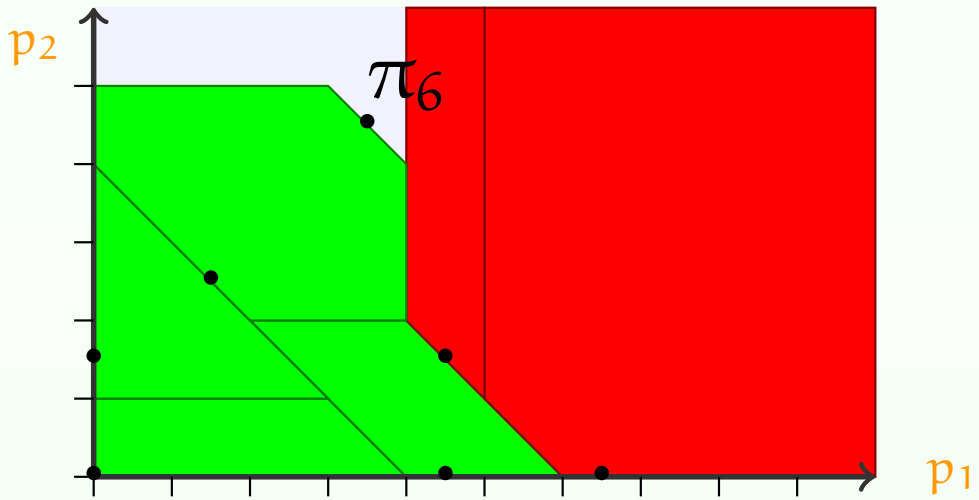

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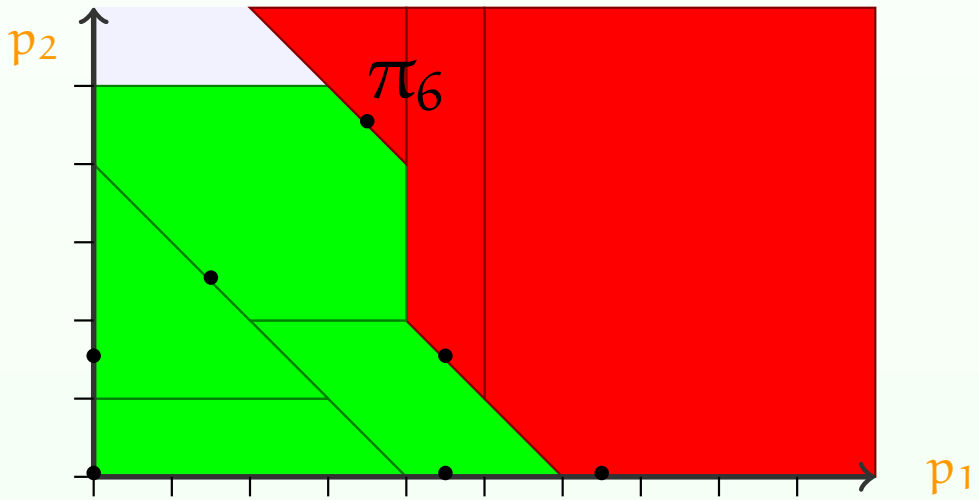

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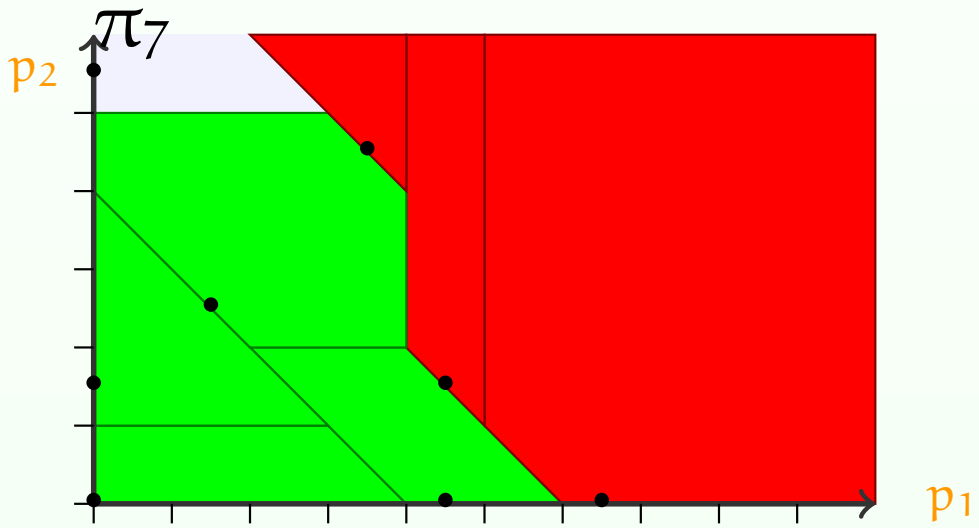

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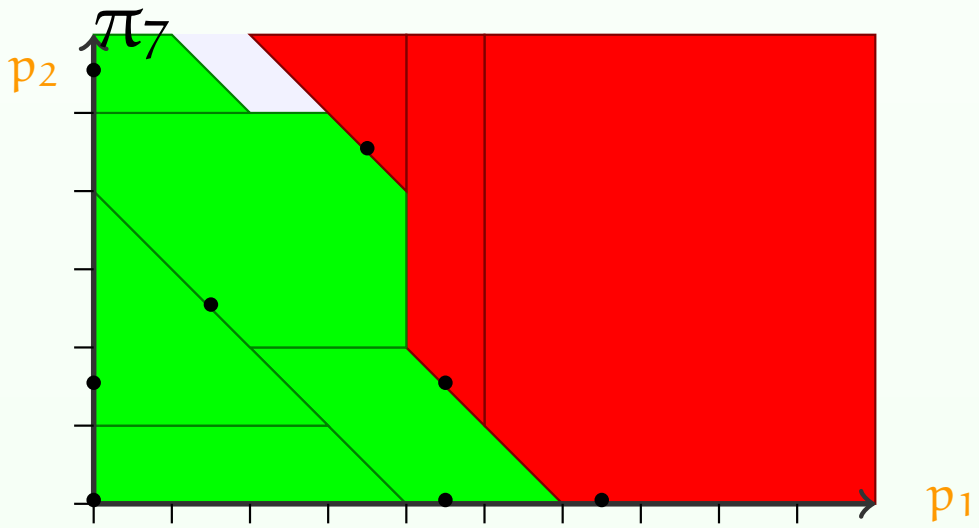

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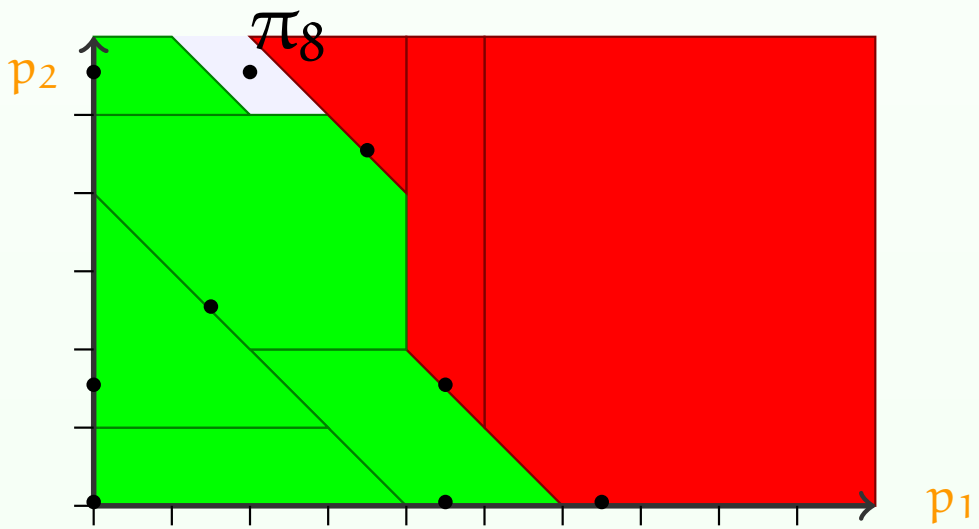

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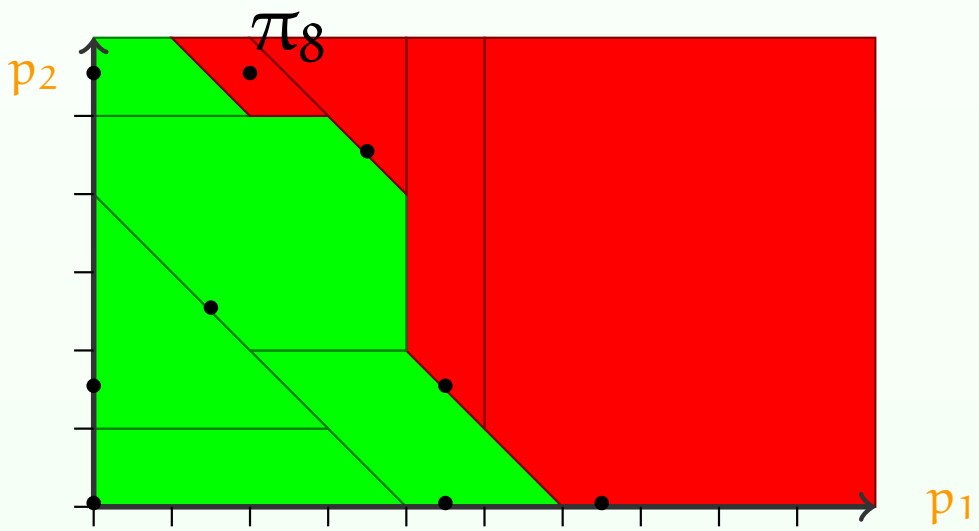

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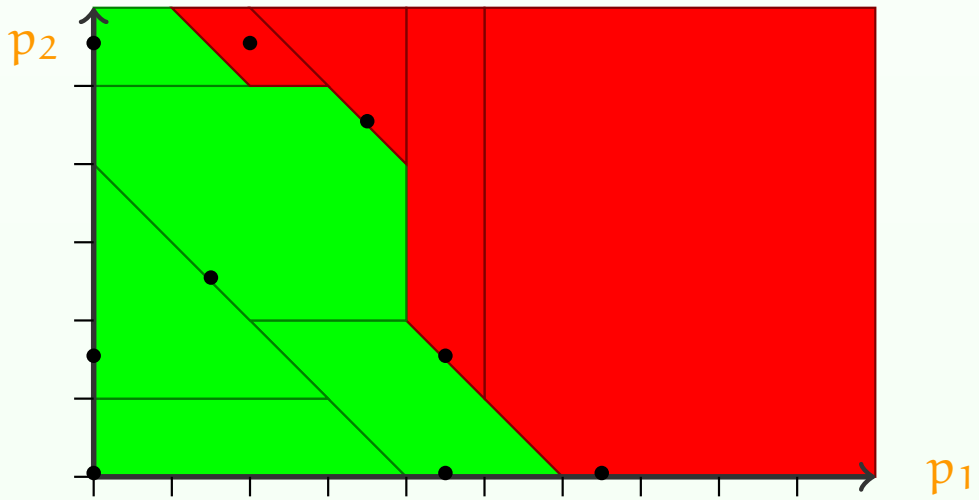


Outline

Comparison

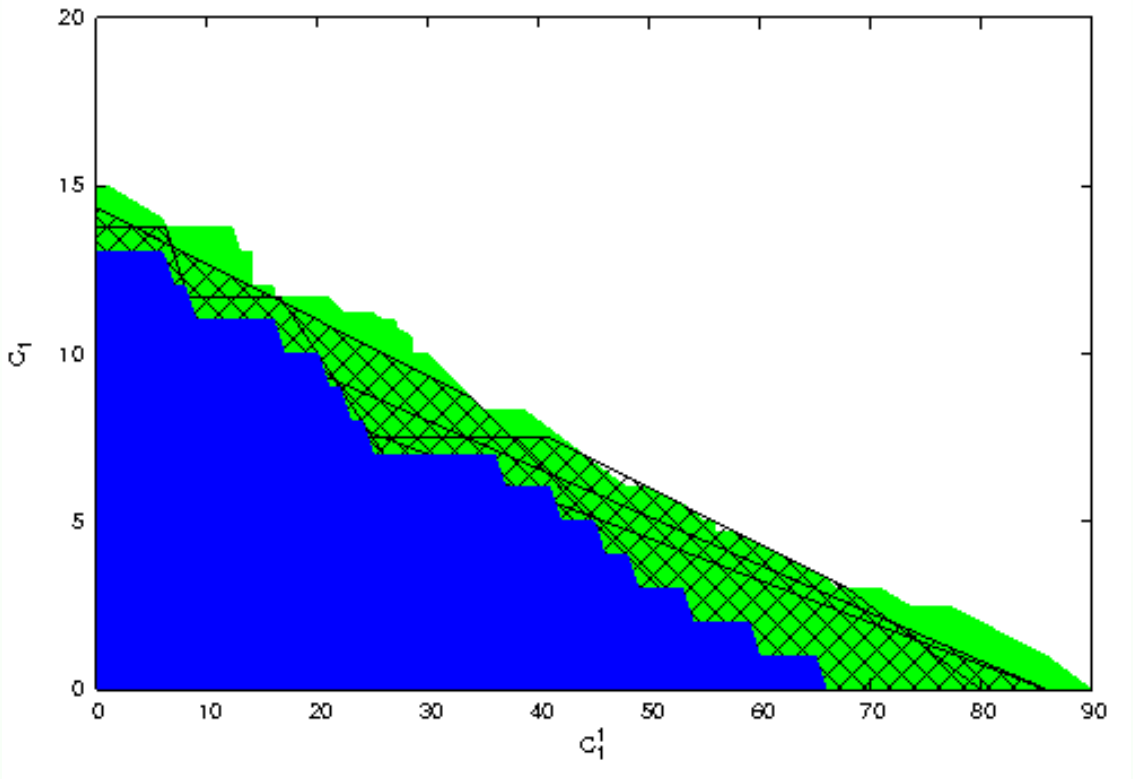
- We evaluate the effectiveness and the running time of the two proposed tools on two case studies.
  - Inverse method implemented in IMITATOR
  - Extension to the holistic analysis implemented in RTSCAN
- As a baseline comparison, we choose to also run the same kind of analysis using MAST [González Harbour et al., 2001]
  - Implements classical holistic analysis
  - Parametric analysis realized by computing schedulability for each possible combination of the values of parameters

First Case Study

- 2 processing nodes (1, 3)
- connected by a single bus (2)
- One pipeline with three tasks and 2 messages
- three other independent tasks (pipelines of length 1)

Pipeline/Task	T	D_{e2e}	Tasks	C	Priority	Processor
τ_1	20	20	-	free	9	1
P^1	150	150	τ^1_1	free	3	1
			τ^1_2	10	9	2
			τ^1_3	8	5	3
			τ^1_4	15	2	2
			τ^1_5	25	2	1
τ_2	30	30	-	6	9	3
τ_3	200	200	-	40	2	3

Results



Time for computing the region:

- RTScan(hatched): 0.27s
- Mast (blue, below): 7s
- Imitator (green, above): 19min42

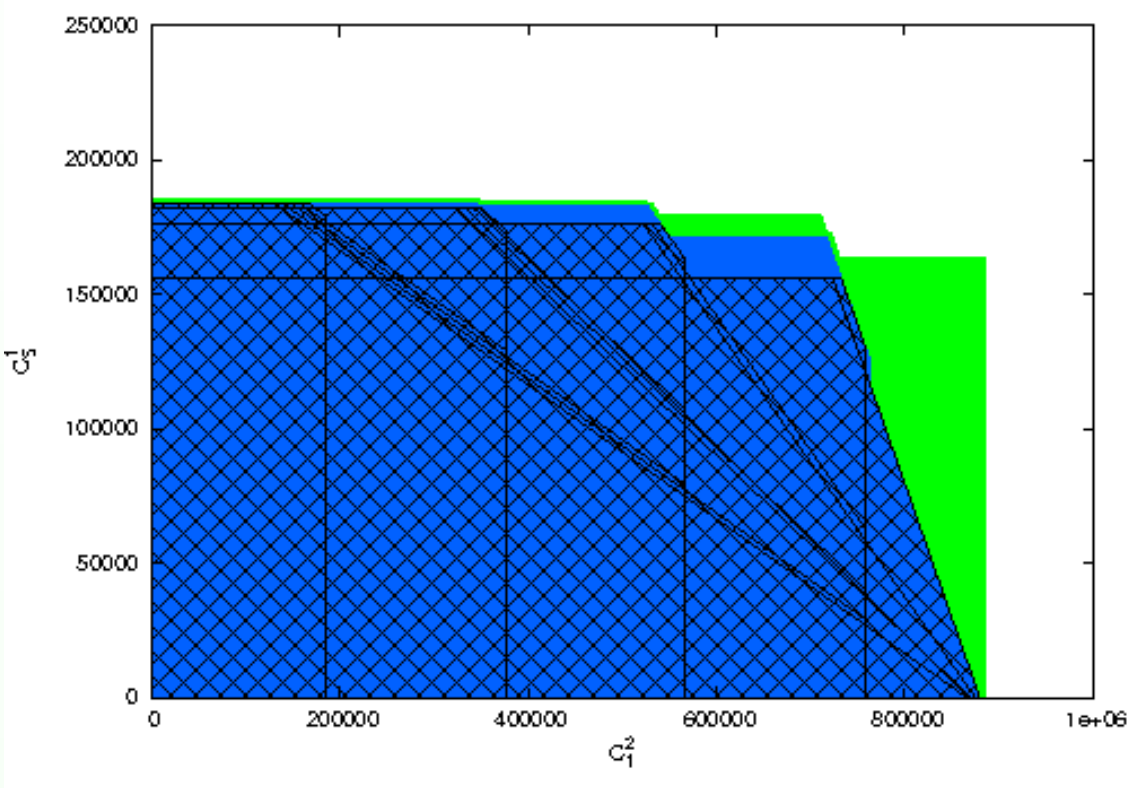
Second Case Study

- 3 processing nodes (1, 3, 4)
- connected by one bus (2)
- Two pipelines

Pipeline	T	D_{e2e}	Tasks	C	Priority	Processor
P^1	200 (30)	200	τ_1^1	4,546	10	1
			τ_2^1	445	10	2
			τ_3^1	9,091	10	4
			τ_4^1	445	9	2
			τ_5^1	free	9	1
P^2	3,000	1,000	τ_1^2	free	9	4
			τ_2^2	889	8	2
			τ_3^2	44,248	10	3
			τ_4^2	889	7	2
			τ_5^2	22,728	8	1

- Periods are in milliseconds, computation times are in microseconds

Results



Time for computing the region:

- RTScan: 0.47s
- Mast: 40min13 (with step of 10µs)
- Imitator: 2h08

Comments

- RTScan: very fast, but uses a lot of memory
  - A test with deadline larger than period easily runs out of memory (on a 8GiB PC)
  - Due to the non-convex nature of the problem
- Mast: no memory problem
  - However, not fully parametric
  - Depends on the tick size (i.e., precision)
- Imitator
  - Each "tile" is analyzed in a few seconds
  - However, a very long time is spent in moving to the next tile
  - We believe that there is much space for improvement here

Outline

Perspectives

- We are confident that the current methodology can be improved and be made more scalable
- Possible extensions:
  - Combine RTSCAN with IMITATOR
    - The first one could find a first approximate solution
    - The second one may refine the initial region by working on the frontier
  - Work on the tile exploration algorithm to improve performance

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Additional explanation

Semantics of Parametric Timed Automata

- State of a PTA: pair (l, C), where
  - l is a location (e.g., ),
  - C is a constraint (conjunction of inequalities) over X and P

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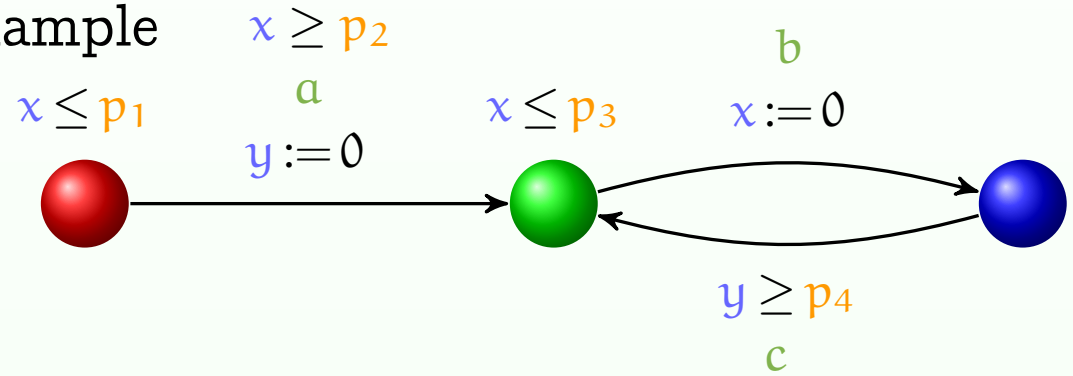
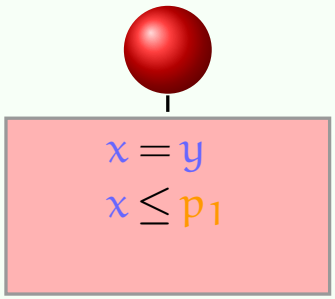
- Example



  - Possible path for this PTA



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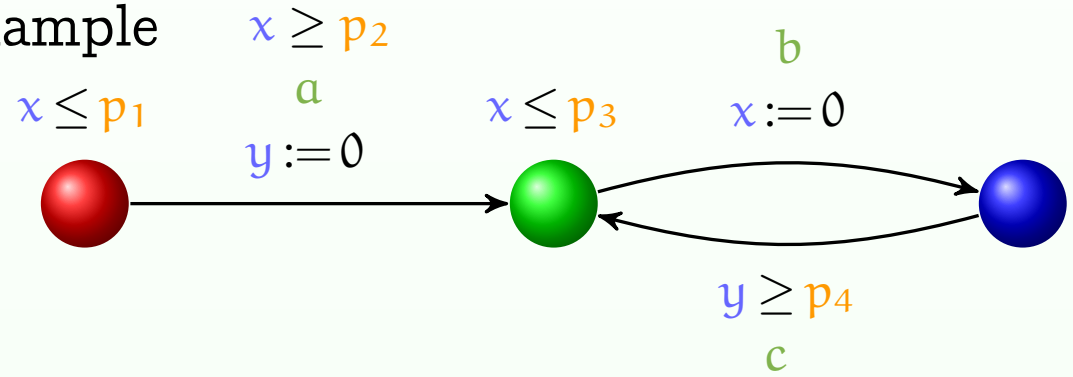

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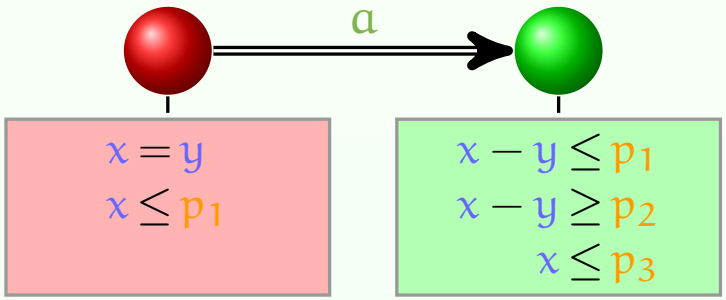

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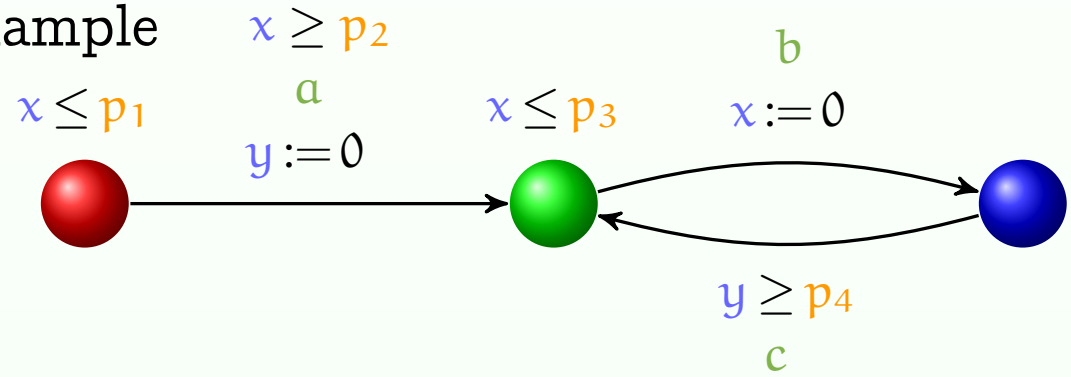

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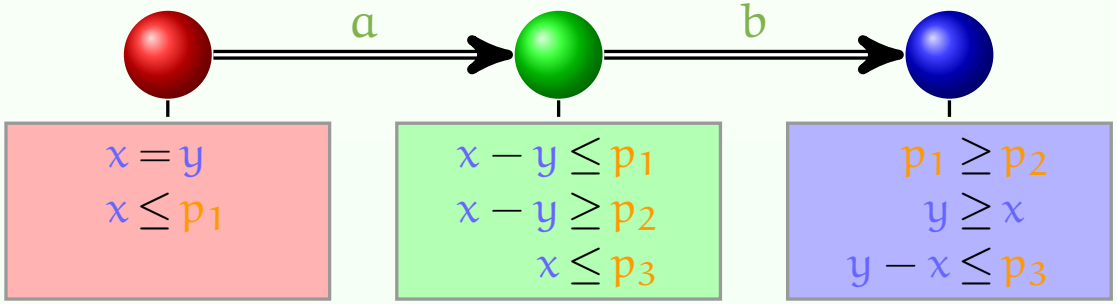


The Inverse Method: Algorithm

Algorithm 1: $IM(\mathcal{A}, \pi)$

input : PTA $\mathcal{A}$ of initial state s_0, parameter valuation π
output: Constraint K over the parameters

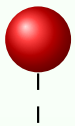
1 $i \leftarrow 0$; $K_c \leftarrow \texttt{true}$; $S_{new} \leftarrow \{s_0\}$; $S \leftarrow \{\}$
2 **while** `true` **do**
3 **while** *there are* π*-incompatible states in* S_{new} **do**
4 Select a π-incompatible state (l, C) of S_{new} (i.e., s.t. $\pi \not\models C$) ;
5 Select a π-incompatible J in $C{\downarrow_P}$ (i.e., s.t. $\pi \not\models J$) ;
6 $K_c \leftarrow K_c \wedge \neg J$; $S \leftarrow \bigcup_{j=0}^{i-1} Post^j_{\mathcal{A}(K_c)}(\{s_0\})$; $S_{new} \leftarrow Post_{\mathcal{A}(K_c)}(S)$;
7 **if** $S_{new} \sqsubseteq S$ **then return** $K \leftarrow \bigcap_{(l,C)\in S} C{\downarrow_P}$
8 $i \leftarrow i+1$; $S \leftarrow S \cup S_{new}$; $S_{new} \leftarrow Post_{\mathcal{A}(K_c)}(S)$

The Inverse Method: Example

π_0 :

$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

$K = \texttt{true}$



$T_{Setup} \leq T_{LO}$

Example of a flip-flop circuit [Clarisó and Cortadella, 2007]

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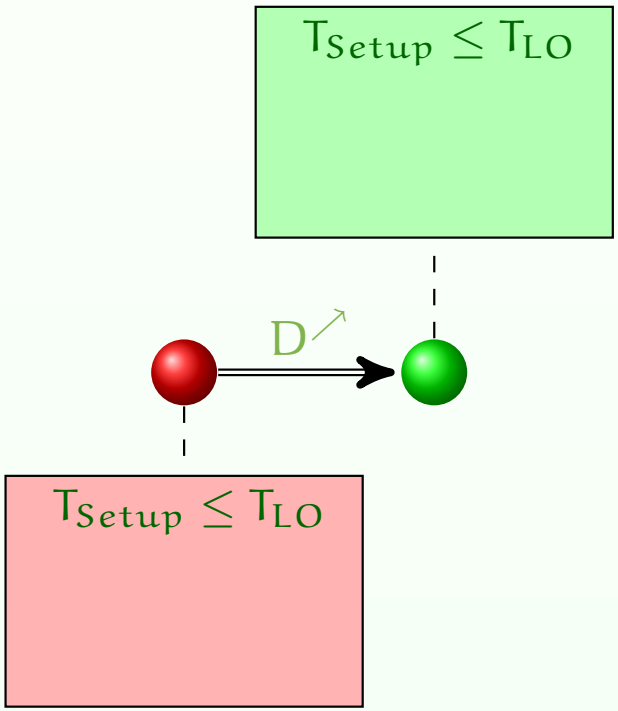

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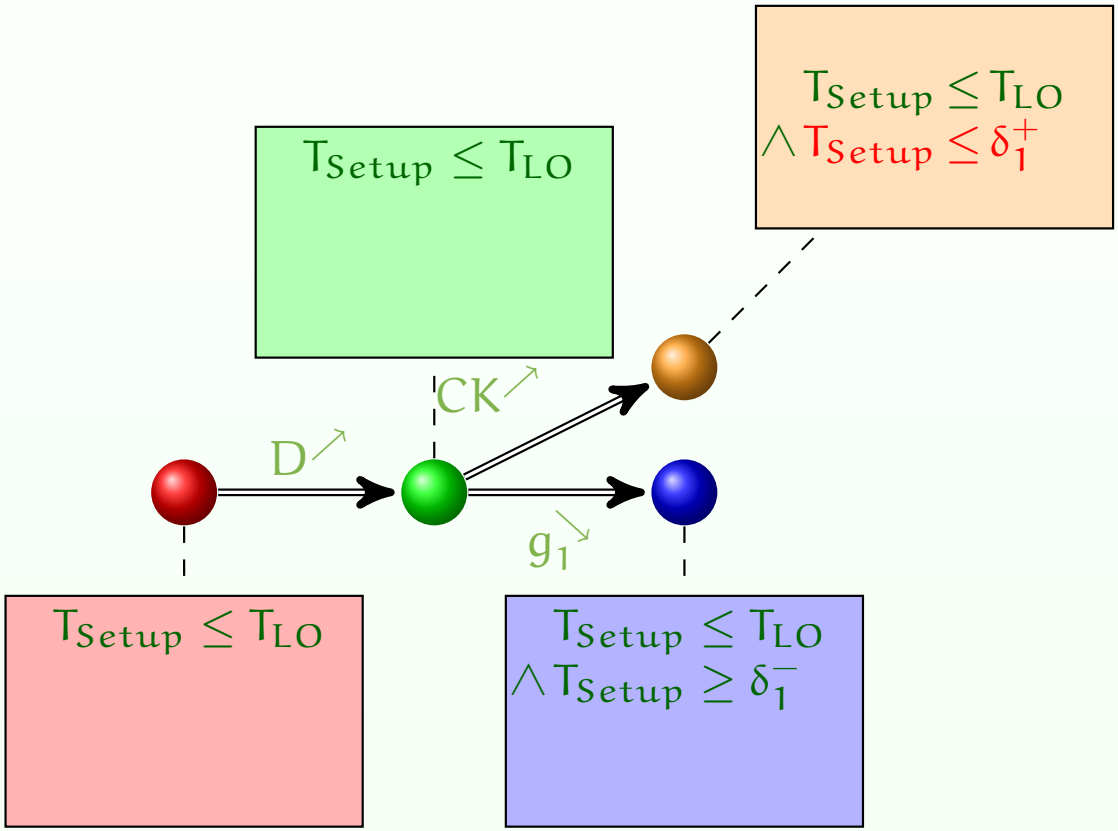

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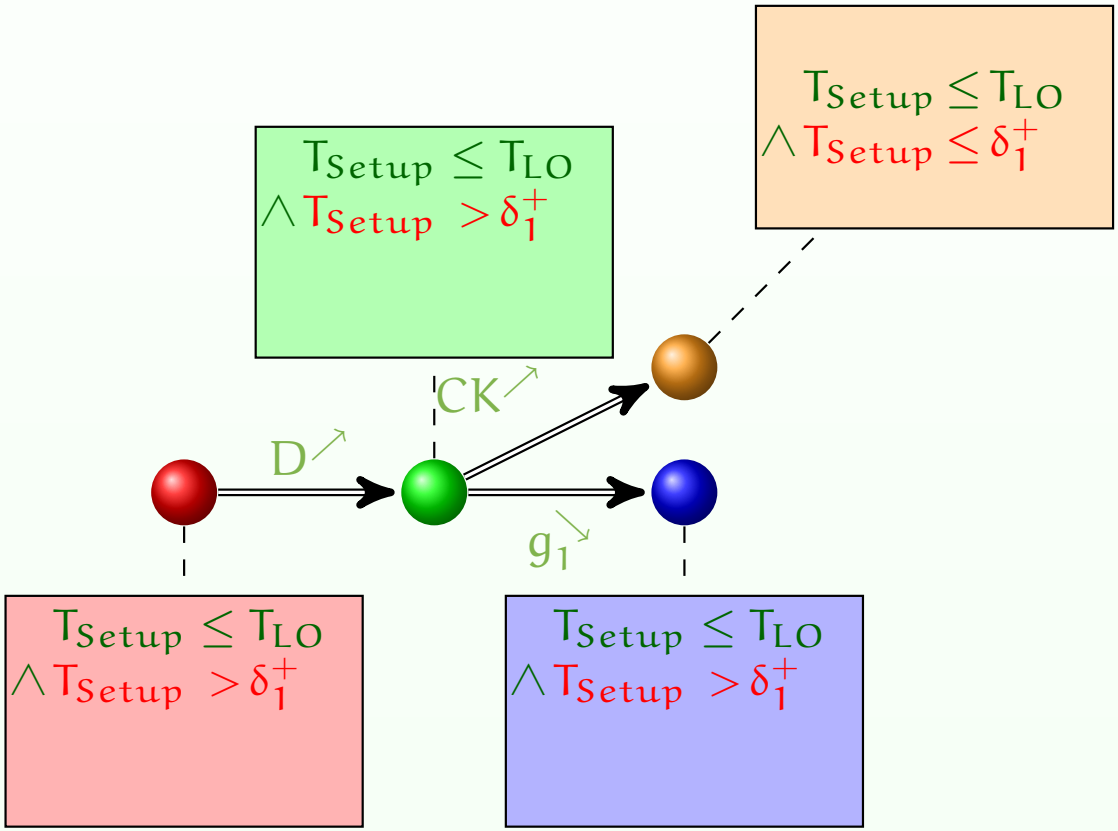
$K = T_{Setup} > \delta_1^+$



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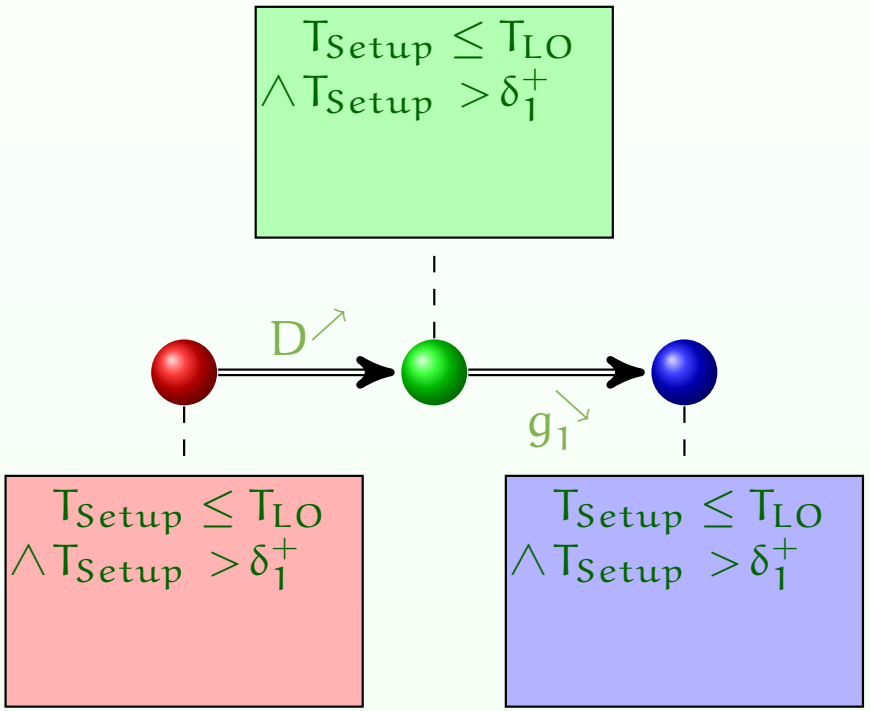

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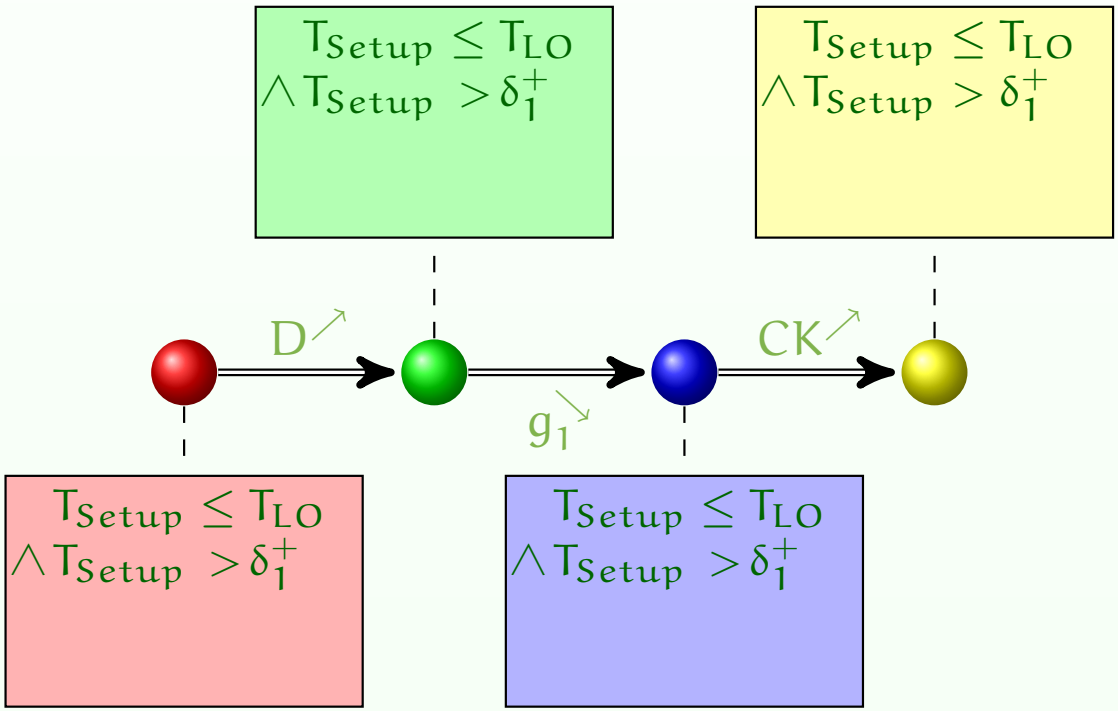

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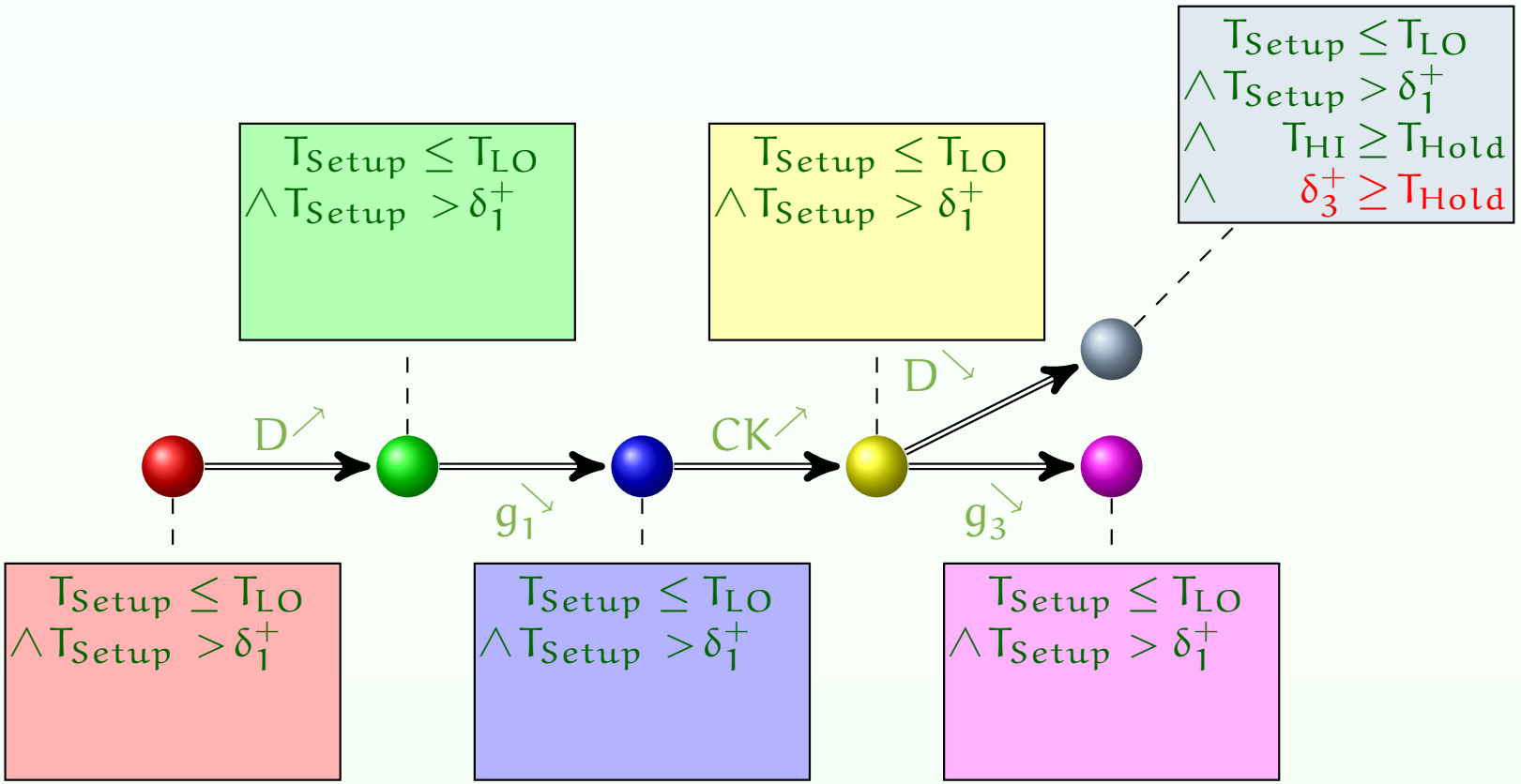

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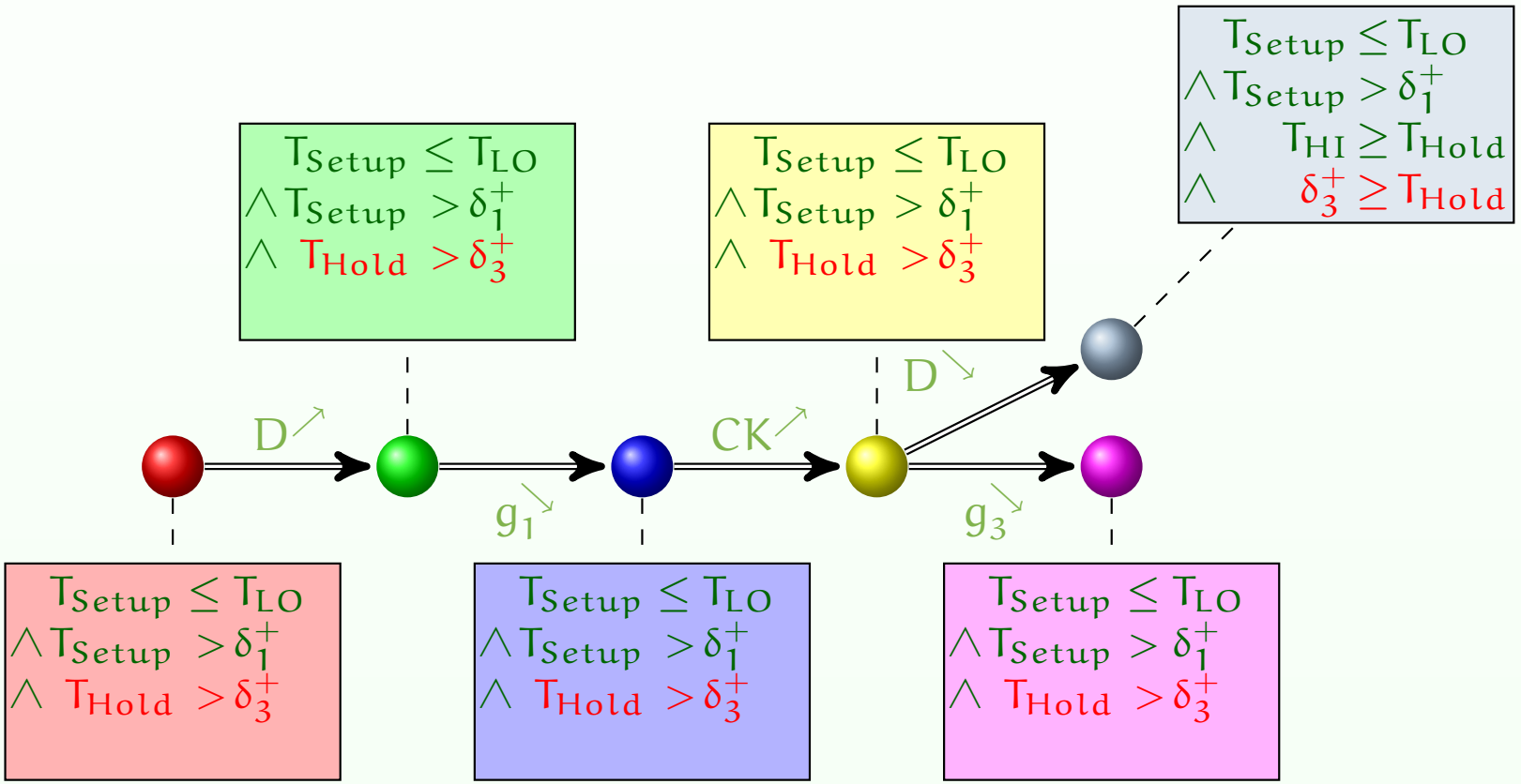
$K = T_{Setup} > \delta_1^+ \wedge T_{Hold} > \delta_3^+$



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$T_{Setup} > \delta_1^+$
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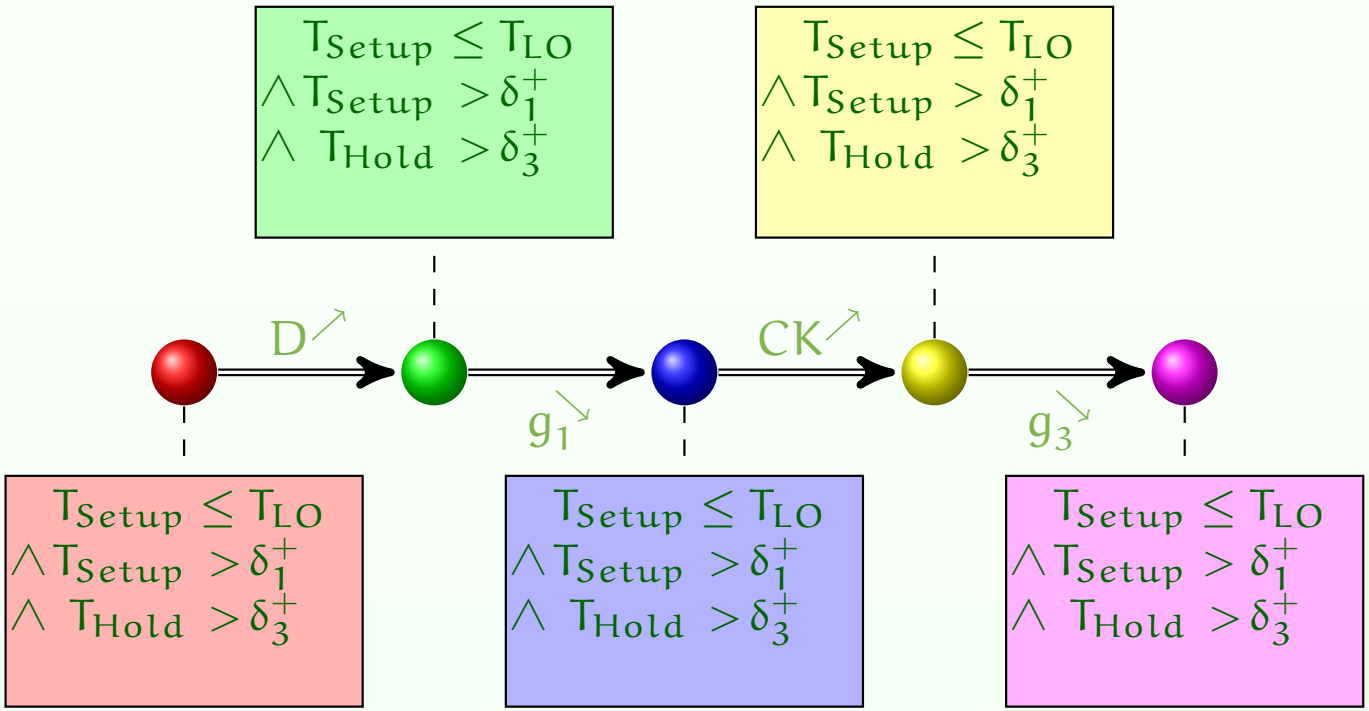

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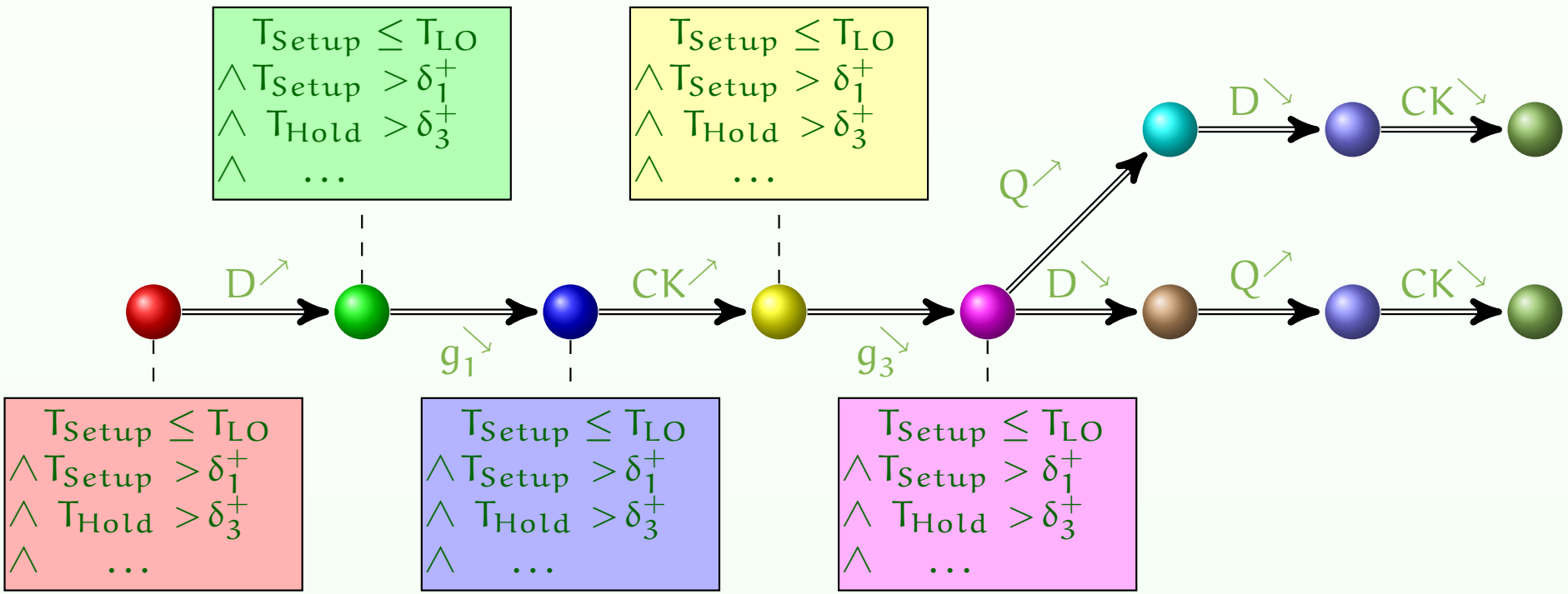
$$
\begin{aligned}
K = \quad & T_{Setup} > \delta_1^+ \wedge \delta_3^+ + \delta_4^+ \geq T_{Hold} \\
\wedge \; & T_{Hold} > \delta_3^+ \wedge \delta_3^+ + \delta_4^+ < T_{HI} \\
\wedge \; & T_{Setup} \leq T_{LO} \wedge \delta_3^- + \delta_4^- \leq T_{Hold} \\
\wedge \; & \delta_1^- > 0
\end{aligned}
$$



Example of a flip-flop circuit [Clarisó and Cortadella, 2007]

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